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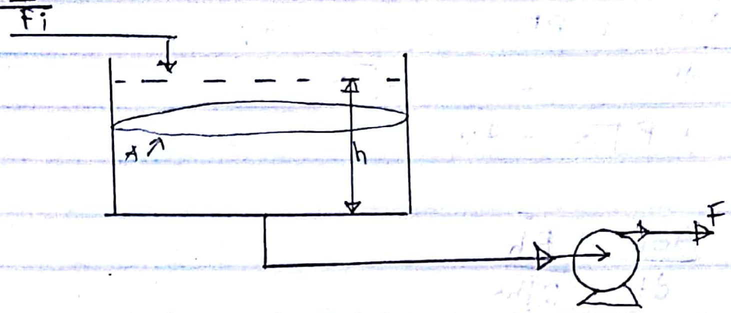
13/ENG01/007
Chemical Engineering
Process Dynamics And Control Assignment
Question 1



Cross-sectional area, A
Inlet flow rate, F_i
Exit flow rate, F

a) To derive its theoretical Dynamic Model, we take model of the system

$V = Ah$

Given $F = \beta\sqrt{h}$

$$\frac{dv}{dt} = F_i - F$$

$$\frac{d(Ah)}{dt} = F_i - \beta\sqrt{h}$$

$$\frac{Adh}{dt} = F_i - \beta\sqrt{h}$$

$$\frac{Adh_s}{dt} = F_{is} - \beta\sqrt{h_s}$$

$$F_{is} = \frac{Adh_s}{dt} + \beta\sqrt{h_s}$$

b) To find its transfer function model

$\beta\sqrt{h}$ is not linear, therefore linearizing $\beta\sqrt{h}$ using the general method for single variable linearization

$$f(x) = F(x_s) + \frac{df}{dx}\Big|_{x=x_s}(x - x_s) \qquad x = \beta\sqrt{h}$$

$$\therefore f(\beta\sqrt{h}) = \beta\sqrt{h_s} + \frac{1}{2}\frac{\beta}{\sqrt{h_s}}(h - h_s) \quad \text{-- - ..}$$

Therefore putting the above equation into the dynamic Model Equation

$$A\frac{dh}{dt} = F_i - \left(\beta\sqrt{h_s} + \frac{\beta}{2\sqrt{h_s}}(h-h_s)\right)$$

$$A\frac{dh}{dt} + \frac{\beta\sqrt{h_s}}{2} + \frac{\beta}{2\sqrt{h_s}}(h-h_s) = F_i$$

To get the deviation variable form of the model

$$A\frac{dh}{dt} + \frac{\beta\sqrt{h_s}}{2} + \frac{\beta}{2\sqrt{h_s}} = F_i$$

$$A\frac{dh_s}{dt} + \beta\sqrt{h_s} = F_{is}$$

$$\bar{F_i} = A\frac{d\bar{h}}{dt} + \frac{\beta\bar{h}}{2\sqrt{h_s}}$$

Taking Laplace of Both sides

$$A\mathcal{L}^{-1}\left[\frac{d\bar{h}}{dt}\right] + \frac{\beta}{2\sqrt{h_s}}\mathcal{L}^{-1}[h'] = \mathcal{L}^{-1}[\bar{F_i}]$$

$$A[s\bar{h} - \bar{h}(0)] + \frac{\beta\bar{h}(s)}{2\sqrt{h_s}} = \bar{F_i}(s)$$

$$As\bar{h}(s) + \frac{\beta}{2\sqrt{h_s}}\bar{h}(s) = \bar{F_i}(s)$$

$$G_p(s) = \frac{h(s)}{F_i(s)} = \frac{\text{Output}}{\text{Input}}$$

$$G_p(s) = \bar{h}_s\left[As + \frac{\beta}{2\sqrt{h_s}}\right] = \bar{F_i}(s)$$

$$G_p(s) = \frac{\bar{h}(s)}{\bar{F_i}(s)} = \frac{1}{\left[As + \frac{\beta}{2\sqrt{h_s}}\right]}$$

$$G_p(s) = \frac{1}{\frac{As}{1} + \frac{\beta}{2\sqrt{h_s}}} \rightarrow G_p(s) = \frac{2\sqrt{h_s}}{2A\sqrt{h_s}\,s + \beta}$$

c) $A = 2.5m^2 \quad h_s = 4m \quad \beta = 2\frac{m}{min}$

$$G_p(s) = \frac{2\sqrt{4}}{2(2.5)(\sqrt{4}s)+2}$$

$$= \frac{2\times 2}{2(2.5)(\sqrt{4})s+2}$$

$$= \frac{4}{10s+2} =$$

$$G_p(s) = \frac{2}{5s+1} = \frac{2}{5s+1}$$

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Question 2 [Manually Obtaining Servo (Set-Point Tracking)]

Transfer function = $G_p(s) = \frac{2}{5s+1}$,

$G_f(s) = G_m(s) = 1$, $k_c = 1.5$

With A 3 step Unit Change

$$y(s) = \frac{G_p(s)\, G_f(s)\, G_c(s)}{1 + G_p(s)\, G_f(s)\, G_c(s)\, G_m(s)} \cdot y_s P(s)$$

$$= \frac{G_p(s)\, G_c(s)}{1 + G_p(s)\, G_c(s)} \div y_s P(s)$$

$$y(s) = \frac{\frac{2}{5s+1} \cdot 1.5}{1 + \frac{2}{5s+1} \cdot 1.5} \div \frac{3}{s} \rightarrow y(s) = \frac{\frac{3}{5s+1}}{1 + \frac{3}{5s+1}} \div \frac{3}{s}$$

$$y(s) = \frac{\frac{3}{5s+1}}{\frac{5s+1+3}{5s+1}} \div \frac{3}{s} \rightarrow y(s) = \frac{\frac{3}{5s+1}}{\frac{5s+4}{5s+1}} \div \frac{3}{s}$$

$$= \frac{3}{5s+1} \div \frac{5s+4}{5s+1} \div \frac{3}{s}$$

$$= \frac{3}{5s+1} \times \frac{5s+1}{5s+4} \cdot \frac{3}{s}$$

$$= \frac{3}{5s+4} \cdot \frac{3}{s} = \frac{9}{s(5s+4)}$$ Using Partial fractions

$$y(s) = \frac{9}{s(5s+4)} = \frac{A}{s} + \frac{B}{5s+4}$$

$$= \frac{A(5s+4) + Bs}{s(5s+4)}$$

$$9 = A(5s+4) + Bs$$

When $s = 0$

$$A(5(0)+4) + B(0)$$

$$9 = 4A$$

$$A = 9/4 = 2.25$$

When $s = -4/5$

$$9 = A\left[5\left(-\frac{4}{5}\right)+4\right] - \frac{4B}{5}$$

$$9 = A(0) - \frac{4B}{5}$$

$$9 = \frac{-4B}{5}$$

$$45 = -4B$$

$$B = \frac{-45}{4}$$

$$\therefore \frac{9}{s(5s+4)} = \frac{9}{4s} - \frac{45}{4(5s+4)}$$

$$\mathcal{L}^{-1}\left(\frac{9}{4s}\right) - \mathcal{L}^{-1}\left[\frac{45}{20(s+4/5)}\right]$$

$$\frac{9}{4}\mathcal{L}^{-1}\left(\frac{1}{s}\right) - \frac{45}{20}\mathcal{L}^{-1}\left[\frac{1}{s+4/5}\right]$$

$$y(t) = \frac{9}{4} - \frac{45}{20}e^{-4/5t}$$

$$= \frac{9}{4} - \frac{9}{4}e^{-4/5t}$$

$$y(t) = \frac{9}{4}\left(1 - e^{-4/5t}\right)$$