Imam A. Sharafudeen
16/Eng06/035

Assignment

The parameter equation of a curve area as given as equation (1) and (2)

$$x = \cos t + t\sin t \qquad (1)$$
$$y = \sin t - t\cos t \qquad (2)$$

In terms of t, determine

a) an expression for the radius of curvature (R)

$$\Rightarrow R = \frac{\left[1 + \left[\frac{dy}{dx}\right]^2\right]^{3/2}}{d^2y/dx^2}$$

$$x = \cos t + t\sin t$$
$$dx/dt = -\sin t + t\cos t + \sin t$$
$$dx/dt = +t\cos t$$

$$y = \sin t - t\cos t$$
$$dy/dt = \cos t - (-t\sin t + \cos t)$$
$$= \cos t + t\sin t - \cos t$$
$$= t\sin t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$
$$= t\sin t \times \frac{1}{t\cos t} = \frac{\cancel{t}\sin t}{\cancel{t}\cos t} = \frac{\sin t}{\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$
$$= \frac{d}{dt}\left(\frac{dy}{dx}\right) \times \frac{dt}{dx}$$
$$= \frac{d}{dt}\left(\frac{\sin t}{\cos t}\right) \times \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{v\,du/dx - u\,dv/dx}{v^2}$$

where $u = \cos t$

$du = \sin t$

$u = \sin t$

$$\frac{d^2y}{dx^2} = \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{dt}{dx}$$

recall, $\because \cos^2 t + \sin^2 t = 1$

$$\frac{1}{\cos^2 t} \times \frac{1}{t\cos t} = \frac{1}{t\cos^3 t}$$

Also recall that

$$R = \frac{\left[1 + \left[dy/dx\right]^2\right]^{3/2}}{d^2y/dx^2}$$

$$R = \frac{\left[1 + (\sin t/\cos t)^2\right]^{3/2}}{1/\cos^3 t}$$

$$R = (1 + \sin 2t/\cos^2 t)^{3/2} \times t\cos 3t/1$$

$$R = (\cos^2 t + \sin x)/(\cos^2 t) \times t\cos^3 t$$

$$R = \frac{1}{(\cos^3 t)^{3/2}} \times t\cos^3 t = \frac{1}{(\sqrt{\cos^2 t})^3} \times t\cos^3 t$$

$$R = \frac{t\cos^3 t}{\cos^3 t} = t$$

$\therefore$ the radius of curvature is t

b) Expressions for the co-ordinates (h, k) of the centre of curvature

$\Rightarrow h = x_1 - R\sin\theta$ (1)

$k = y_1 + R\cos\theta$ (2)

$R = t t = \theta$

$\therefore x_c = \cos t + \sin t$

$y_c = \sin t + t\cos t$

substituting x, h eqn (1) and y eqn (2)

$h = \cos t + t\sin t - t\sin t$

$h = \cos t$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$

~~the~~ expressions for the coordinates (h, k) of the centre of curvature is $(\cos t, \sin t)$