ENG 221.

Assignment 2 (LMS)

1). If $z = f\left(\frac{y}{x}\right)$ Show that $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = 0$

Soln.

$$\frac{\partial z}{\partial x} = \frac{y}{x^2}; \quad \frac{\partial^2 z}{\partial x^2} = 2x\frac{y}{x^2} = \frac{2y}{x^3}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial z}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{-1}{x^2}.$$

$$\frac{\partial z}{\partial y} = \frac{1}{x}, \quad \frac{\partial^2 z}{\partial y^2} \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \frac{d}{dy}\left(\frac{1}{x}\right) = 0.$$

$$\frac{\partial z}{\partial y \partial x} = -\frac{1}{x^2}$$

$$\therefore x^2\left(\frac{\partial^2 z}{\partial x^2}\right) + 2xy\left(\frac{\partial^2 z}{\partial x y}\right) + y^2\left(\frac{\partial^2 z}{\partial y^2}\right)$$

$$= x^2\left(\frac{2y}{x^3}\right) + 2xy\left(-\frac{1}{x^2}\right) + y^2(0)$$

$$= \frac{2y}{x} - \frac{2y}{x} + 0 = 0 + 0 = 0 \quad (Q.E.D).$$

2). $$z = \frac{a^2}{x^2+y^2-a^2}$$

$z = \frac{a^2}{u}$ Let $u = a^2+y^2-a^2$

then

$$\partial z = \left(\frac{dz}{dx}\right)dx + \left(\frac{dz}{dy}\right)dy$$

$$= \left(\frac{-a^2}{u^2} \times 2x\right)dx + \left(\frac{-a^2}{u^2} \times 2y\right)dy$$

because x and y have the same percentage error

therefore $dx = px$ and $dy = py$.

$$dz = \rho\left(\frac{-a^2}{u^2} \times 2x\right)dx + \rho\left(\frac{-a^2}{u^2} \times 2dy\right)dy.$$

Factorise.

$$dz = -\frac{2a^2}{a^2}\rho(x^2+y^2).$$

Hence

$$\frac{dz}{z} = -\frac{2a^2}{u^2}\rho(x^2+y^2) \times \frac{\cancel{a}}{a^2} \quad \left(\text{Recall } z = \frac{a^2}{u}\right).$$

$$\therefore \frac{dz}{z} = -\frac{2p}{u}(u+a^2) \qquad \text{Recall } u = x^2+y^2-a^2$$

$$\therefore x^2+y^2 = u+a^2.$$

$$= -2p\left(1+\frac{a^2}{u}\right). \qquad \text{Recall } z = \frac{a^2}{u}$$

$$\therefore \frac{dz}{z} = -2p(1+z) \quad (\text{Percentage Error})$$

Q.E.D