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15/ENG07/031
PETROLEUM ENGINEERING
ENG 381 ASSIGNMENT III

1 If $y = e^{x^2+x}$, Show that $y'' = y'(2x+1) + 2y$ and hence prove that
$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Soln

a $y = e^{x^2+x}$

let $u = e^{x^2+x}$; $u' = (2x+1)e^{x^2+x}$; $u'' = (2x+1)(2x+1)e^{x^2+x}$

$u^n = (2x+1)^n e^{x^2+x}$

and $v = 1$; $v' = 0$

$$y^n = u^n v^0 + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \dots$$

$$y^n = (2x+1)^n e^{x^2+x}(1) + n(2x+1)^{n-1}(0)$$

$$\therefore y^n = (2x+1)^n e^{x^2+x}$$

Let $n = 1$

$$y^{(1)} = (2x+1)^1 e^{x^2+x}$$

$u = e^{x^2+x}$; $u' = (2x+1)e^{x^2+x}$; $u'' = (2x+1)(2x+1)e^{x^2+x}$

$u^n = (2x+1)^n e^{x^2+x}$

$V = 2x+1$; $V' = 2$; $V'' = 0$

$$\Rightarrow y^{1(n)} = u^n v^0 + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} V''$$

$$y^{1(n)} = (2x+1)^n e^{x^2+x}(2x+1) + n(2x+1)^{n-1} e^{x^2+x}(2) + \frac{n(n-1)}{1\cdot 2}(2x+1)^{n-2}(0)$$

$$y^{1(n)} = (2x+1)(2x+1)^n e^{x^2+x} + 2n(2x+1)^{n-1} e^{x^2+x} + 0$$

Where $n = 1$ (Differentiating y^{1n})

$$y'' = (2x+1)(2x+1)^1 e^{x^2+x} + 2(1)(2x+1)^{1-1} e^{x^2+x}$$

$$y'' = (2x+1)(2x+1)e^{x^2+x} + 2(2x+1)^0 e^{x^2+x}$$

$$y'' = (2x+1)(2x+1)e^{x^2+x} + 2(e^{x^2+x})$$

Substitute $y = e^{x^2+x}$ and $y' = (2x+1)^1 e^{x^2+x}$ in y''

$$y'' = (2x+1)y' + 2y$$

$$\therefore y'' = y'(2x+1) + 2y$$

b To proove that $y^{(n+2)} = (2x+1)y^{n+1} + 2(n+1)y^n$

$$y'' = y'(2x+1) + 2y$$

Let $w = y''$

$u = y''$; $u' = y'''$; $u^n = y^{n+2}$

$V = 1$; $V' = 0$

$$w^n = u^n v^0 + n u^{n-1} v'$$

$$w^n = y^{n+2}(1) + n(y^{n-1})(0)$$

$w^n = y^{n+2}$

Let $w = y'(2x+1)$

$u = y'$; $u' = y''$; $u^n = y^{n+1}$

$v = 2x+1$; $v' = 2$; $v'' = 0$

$\therefore w^n = u^n v^0 + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v''$

$w^n = y^{n+1}(2x+1) + n(y^n)(2) + \frac{n(n-1)}{1\cdot 2}(y^{n-1})(0)$

$w^n = y^{n+1}(2x+1) + 2ny^n$

Let $w = 2y$

$u = y$; $u^n = y^n$

$v = 2$; $v' = 0$

$w^n = u^n v^0 + n u^{n-1} v'$

$w^n = y^n(2) + n(y^{n-1})(0)$

$w^n = 2y^n$

$\therefore y'' = y'(2x+1) + 2y$

$\Rightarrow y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + (2n+2)y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$ QED

2. Using the leibnitz theorem

i. $y = x^3 e^{4x}$, determine $y^{(5)}$

$u = e^{4x}$; $v = x^3$

$u^n = 4^n e^{4x}$; $v' = 3x^2$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$; $v'' = 6x$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$; $v''' = 6$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$; $v^{iv} = 0$

$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v'' + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v'''$

$+ \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} v^{iv}$

$\therefore y^n = 4^n e^{4x} \cdot x^3 + n 4^{(n-1)} e^{4x}(3x^2) + \frac{n(n-1)}{2!} 4^{(n-2)} e^{4x}(6x)$

$+ \frac{n(n-1)(n-2)}{3\times 2} 4^{(n-3)} e^{4x}(6) + \frac{n(n-1)(n-2)(n-3)}{4\times 3\times 2} \times 4^{(n-4)} e^{4x}(0)$

$y^n = 4^n e^{4x} \cdot x^3 + n 4^{(n-1)} e^{4x}(3x^2) + n(n-1) 4^{(n-2)} e^{4x}(3x) +$
$n(n-1)(n-2) 4^{(n-3)} e^{4x}$

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$$y^5 = 4^5 e^{4x} \cdot x^3 + (5)4^{(5-1)} e^{4x}(3x^2) + 5(5-1)\,4^{(5-2)} e^{4x}(3x)$$
$$+\ 5(5-1)(5-2)\,4^{(5-3)} e^{4x}$$
$$y^5 = 1024\, e^{4x} \cdot x^3 + 5(256)\, e^{4x}(3x^2) + 20(64)\, e^{4x}(3x)$$
$$+\ 60(16)\, e^{4x}$$
$$y^5 = 1024\, x^3 e^{4x} + 3840\, x^2 e^{4x} + 3840\, x e^{4x} + 960\, e^{4x}$$
$$y^5 = (1024x^3 + 3840x^2 + 3840x + 960)\, e^{4x}$$

ii $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$; Show that

$$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$$

Sln

$$x^2 y'' + xy' + y = 0$$

Let $w = x^2 y''$

$u = y''$; $u' = y'''$; $u'' = y^{iv}$; $u^n = y^{(n+2)}$

$V = x^2$; $V' = 2x$; $V'' = 2$; $V''' = 0$

$$w^n = u^n v^0 + n u^{(n-1)} V' + \frac{n(n-1)}{2!} U^{(n-2)} V'' + \frac{n(n-1)(n-2)}{3!} U^{(n-3)} V^{iv}.$$
$$w^n = y^{(n+2)} x^2 + n y^{n+1}(2x) + \frac{n(n-1)}{2} y^n(2) + \frac{n(n-1)(n-2)}{6} y^{n-1}(0)$$
$$w^n = y^{(n+2)} x^2 + 2nx\,y^{(n+1)} + n(n-1)y^n$$

Let $w = xy'$

$V = x$; $V' = 1$; $V'' = 0$

$U = y'$; $U' = y''$; $U^n = y^{(n+1)}$

$$w^n = U^n V^0 + n U^{(n-1)} V' + \frac{n(n-1)}{2!} U^{(n-2)} V''$$
$$w^n = y^{(n+1)} x + n y^n + \frac{n(n-1)}{2} y^{n-1}(0)$$
$$w^n = x y^{(n+1)} + n y^n$$

Let $w = y$

$u = y$; $u^n = y^n$

$\therefore w^n = y^n$

$$y^{(n+2)} x^2 + 2nx\,y^{(n+1)} + n(n-1)y^n + xy^{(n+1)} + ny^n + y^n = 0$$
$$y^{(n+2)} x^2 + (2nx + x)y^{(n+1)} + (n(n-1) + n + 1)y^n = 0$$
$$x^2 y^{(n+2)} + (2nx + x)y^{(n+1)} + (n^2 - n + n + 1)y^n = 0$$
$$x^2 y^{(n+2)} + (2n+1)x\,y^{(n+1)} + (n^2+1)y^n = 0$$