15/Eng07/022
Hamzat-odusoga-Kaindeyoo
Petorleum.

If $y = e^{x^2+x}$

$u = x^2+x$

$\frac{du}{dx} = 2x+1$

$y = e^u$

$\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= e^u \times 2x+1$

$2x+1e^u \quad u = x^2+x$

$\frac{dy}{dx} = 2x+1e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2+4x+1e^{x^2+x}$

$\frac{d^2y}{dx} \quad 2e^{x^2+x} + 4x^2 + 4x1 \cdot e^{x^2+x}$

$y^2 = \frac{d^2y}{dx^2} \quad y' = \frac{dy}{dx} \quad y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y'' = y'(2x+1) + 2y$

$y'' = 2e^{x^2+2} + 4x^2+4x+1 \cdot e^{x^2+x}$

$y'(2x+1)(2x+1)(2x+1)e^{x^2+x}$

$= 4x^2+4x+4e^{x^2+x}$

$2y = 2e^{x^2+x}$

$y'(2x+1)+2y = 2e^{x^2+x} + 4x^2+4x+1e^{x^2+x}$

$= 2e^{x^2+2} + 4x^2 + 4x + 1e^{x^2+x}$

$y'' = \underbrace{y'(2x+1)}_{W2} + \underbrace{2y}_{W3}$

$u = y''$ $\quad v = 1$

$u^n = y^{n+2}$ $\quad v = 0$

$y^{n+2} \cdot 1 + 0$

w_2

$u = y'$ $\quad V = 2x+1$

$u^n = y^{n+1}$ $\quad V' = 2$

$y^{n-1} = y^n$ $\quad V = 0$

$y^{n+1}(2x+1 + n(y^n)2 + 0$

w_3

$u = 2y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0$

$2[(y^n \cdot 1) + 0]$

$= 2y^n$

$\cancel{u = 2y}$ $\quad \cancel{v = 1}$ $\quad w_1 = w_2 + w_3$

$\cancel{u^n = y^n}$ $\quad \cancel{v' = 0}$ $\quad y^{n+1}(2x+1) + 2n(y^n + 2y^n$

$\cancel{2[(}$ $\quad = y^{n+1}(2x+1) + 2(n+1)y^n$

2a) using the ~~leibnitz~~ theorem given that $y = x^3 e^{4x}$ determine $y^{(5)}$

Sol

$u = e^{4x}$ $\quad v = x^3$

$y^{(5)} = u^{(5)}v + 5u^{iv}v' + 10u^3v'' + 10u^2v^3 + 5u^i v^4 + uv^5$

$= 4^5 e^{4x} x^3 + 5(4^4 e^{4x} \cdot 3x^2) + 10(4^3 e^{4x} 6x) + 5(4^2 e^{4x} 6) + 0$

$= 1024 e^{4x} x^3 + 1280 e^{4x} 3x^2 + 640 e^{4x} 6x + 80 e^{4x} 6$

$1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x} + 480 e^{4x}$

4) $x^2 \frac{dy^2}{dx^2} + \frac{x dy}{dx} + y = 0$

$x^2 y'' + xy' + y = 0$

$w_1 + w_2 + w_3 = 0$

for w_1

$u = y''$ $\quad v = x^2$

$u^n = y^{n+2}$ $\quad v' = 2x$

$u^{n-1} = y^{n+1}$ $\quad v'' = 2$

$y^{n-2} = y^{n}$ $\quad v''' = 0$

$= y^{(n+2)}(x^2) + n(y^{n+1})\, 2x + n(n-1)y^{n} \cdot x + 0$

$x y^{(n+2)} + 2nx(y^{n+1}) + n(n-1)y^{n}$

for w_2

$u = y^2$ $\quad u = x$

$u_n = y^{n+1}$ $\quad v = 1$

$u_{n-1} = y^{n}$ $\quad v'' = 0$

$y^{n+1} \cdot x + n y^{n} + 0$

for w_3

$u = y$ $\quad v = 1$

$u^{n} = y^{n}$ $\quad v' = 0$

$y^{n} \cdot 1$

$w_1 + w_2 + w_3 = 0$

$x^2 y^{n+2} + 2nx(y^{n+1}) + (n^2 - n)y^{n} + x y^{n+1} + n y^{n} + y^{n}$

$x y^{n+2} + 2nxy^{n+1} + x y^{n+1} + n^2 y^{n} - \cancel{ny^{n}} + \cancel{ny^{n}} + y^{n}$

$x^2 y^{n+2} + (2n + 1)(x y^{n+1}) + (n^2 + 1)y^{n}$