

Name: Awojana Ceciliawati Emmanuel

Dept: Mechanical Engineering

Matric No: 15/ENG06/014

## Assignment II

Course: ENG381 (Engineering Mathematics)

$$1) \frac{d^2y}{de^2} + 4\frac{dy}{de} + 5y = 6\sin e$$

Note: It is a non-homogeneous equation.

∴ General solution = Complementary function + Particular integral

∴ Complementary function:

$$\frac{d^2y}{de^2} + 4\frac{dy}{de} + 5y = 0$$

Auxiliary equation

$$m^2 + 4m + 5 = 0$$

$$m^2 + 4m = -5$$

$$m^2 + 4m + (2)^2 = -5 + (2)^2$$

$$(m+2)^2 = -1$$

$$m+2 = \pm\sqrt{-1}$$

$$m+2 = \pm i$$

$$\therefore m = -2+i \quad \& \quad m = -2-i$$

$$\therefore CF = C_1 e^{(-2+i)e} + C_2 e^{(-2-i)e}$$

$$y_h = C_1 e^{-2e+i e} + C_2 e^{-2e-i e}$$

$$y_h = C_1 e^{-2e} \cdot e^{ie} + C_2 e^{-2e} e^{-ie}$$

$$y_h = e^{-2e} [C_1 e^{ie} + C_2 e^{-ie}]$$

$$y_h = e^{-2e} [A \cos e + B \sin e]$$

$$y_p = A \cos e + B \sin e$$

$$y'_p = -A \sin e + B \cos e$$

$$y''_p = -A \cos e - B \sin e$$

$$-A \cos e - B \sin e + 4[-A \sin e + B \cos e] + 5A \cos e + 5B \sin e = 6 \sin e$$

$$-A \cos e - B \sin e - 4A \sin e + 4B \cos e + 5A \cos e + 5B \sin e = 6 \sin e$$

$$4A \cos e + 4B \sin e - 4A \sin e + 4B \cos e = 6 \sin e$$

$$(4A+4B) \sin e + (4A+4B) \cos e = 6 \sin e$$

$$-4A+4B = 6$$

$$4A+4B = 0$$

$$8B = 6$$

$$\therefore B = \frac{6}{8}$$

$$4A = -4B$$

$$A = -B$$

$$A = -3/4$$

$$y_p = -3/4 \cos \theta + 3/4 \sin \theta$$

$$y = y_h + y_p$$

$$y = e^{-2\theta} [A \cos \theta + B \sin \theta] + 3/4 \sin \theta - 3/4 \cos \theta$$

∴ Steady state equation

$$y' = 0$$

$$\therefore y_p' = 3/4 \cos \theta + 3/4 \sin \theta = 0$$

$$= 3/4 \cos \theta + 3/4 \sin \theta = 0$$

$$3/4 \cos \theta = -3/4 \sin \theta$$

$$\cos \theta = -\sin \theta$$

$$\frac{\sin \theta}{\cos \theta} = \frac{-\cos \theta}{\cos \theta}$$

$$\tan \theta = -1$$

$$\therefore \tan \theta = 1$$

$$2) EI \frac{d^2 y}{dx^2} = \frac{w}{2} (L - xc)^2$$

The auxiliary equation becomes

$$EI m^2 = 0$$

$$m^2 = 0$$

$$m = \pm \sqrt{0}$$

$$m = 0$$

$$\therefore y = e^{0x} [A + Bx]$$

$$y = A + Bx$$

$$y_p = y = Fx^2 + Gx^3 + Hx^4$$

$$\frac{dy}{dx} = 2Fx + 3Gx^2 + 4Hx^3$$

$$\frac{d^2 y}{dx^2} = 2F + 6Gx + 12Hx^2$$

$$EI [2F + 6Gx + 12Hx^2] = \frac{w}{2} (L - xc)^2$$

$$2FEI + 6Gx EI + 12Hx^2 EI = \frac{w}{2} (L - xc)^2$$

$$4FEI + 12Gxc EI + 24Hx^2 EI = w(L - xc)^2$$

$$4FEI + 12EI \alpha C + 24HEI \alpha^2 = w(L^2 - 2L\alpha C + \alpha^2 C^2)$$

$$24HEI = w$$

$$H = \frac{w}{24EI} \quad \dots (i)$$

$$24HEI = w$$

$$12GEI = -2wL$$

$$G = \frac{-2wL}{12EI} = \frac{-wL}{6EI} \quad \dots (ii)$$

$$4FEI = wL^2$$

$$F = \frac{wL^2}{4EI}$$

$$y = \left[ \frac{wL^2}{4EI} \right] \alpha^2 - \left[ \frac{wL}{6EI} \right] \alpha^3 + \left[ \frac{w}{24EI} \right] \alpha^4$$

$$= \frac{wL^2 \alpha^2}{4EI} - \frac{wL \alpha^3}{6EI} + \frac{w \alpha^4}{24EI}$$

$$= \frac{6wL^2 \alpha^2 - 4wL \alpha^3 + w \alpha^4}{24EI}$$

$$G \cdot E = y = A + B\alpha C + \frac{w}{24EI} [6L^2 \alpha^2 - 4L \alpha^3 + \alpha^4]$$

$$\text{at } y=0, \alpha C=0, \frac{dy}{d\alpha C}=0$$

$$0 = A + B(0) + \frac{w}{24EI} [6L^2(0) - 4L(0) + 0]$$

$$A = 0$$

$$\frac{dy}{d\alpha C} = B + \frac{w}{24EI} [12L^2 \alpha - 12L \alpha^2 + 4 \alpha^3]$$

$$0 = B + \frac{w}{24EI} [12(0) - 12(0) + 4(0)]$$

$$B = 0$$

$$y_p = \frac{w}{24EI} [6L^2 \alpha^2 - 4L \alpha^3 + \alpha^4]$$

$$y_p = \frac{w \alpha^2}{24EI} [6L^2 - 4\alpha L + \alpha^2]$$

$$y_p = \frac{w \alpha^2}{24EI} [\alpha^2 - 4\alpha L + 6L^2]$$

when  $\alpha = L$

$$y_p = \frac{wL^2}{24EI} [L^2 - 4L^2 + 6L^2]$$

$$y_p = \frac{wL^2}{24EI} [3L^2]$$

$$y = \frac{wL^4}{8EI}$$