NAME: UNWEBHCHU EMMANUEL OSERENE
MATRIC NO: 15/ENG07/049
DEPARTMENT: PETROLEUM ENGR
COURSE: ENG 381 (ENGINEERING MATHEMATICS III)

Assignment II

i) $d^2y/d\theta^2 + 4\, dy/d\theta + 5y = 6\sin\theta$

Solution

$y'' + 4y' + 5y = 6\sin\theta$

Assume homogeneity

$y'' + 4y' + 5y = 0$

Since $y = e^{kx}$; $y' = ke^{kx}$, $y'' = k^2 e^{kx}$

$k^2 e^{kx} + 4ke^{kx} + 5e^{kx} = 0$

$e^{kx}[k^2 + 4k + 5] = 0$

Since $y = e^{kx} \neq 0$

$k^2 + 4k + 5 = 0$

$k^2 + 4k = -5$

$k^2 + 4k + 4 = -5 + 4$

$(k+2)^2 = -1$

$k + 2 = \pm\sqrt{-1}$

$k_1 = -2 + i$ and $k_2 = -2 - i$

$y_h = e^{-2\theta}(A\cos\theta + B\sin\theta)$

Let $y_p = A\cos\theta + B\sin\theta$

$y_p' = -A\sin\theta + B\cos\theta$

$y_p'' = -A\cos\theta - B\sin\theta$

$-A\cos\theta - B\sin\theta + 4(-A\sin\theta + B\cos\theta) + 5(A\cos\theta + B\sin\theta) = 6\sin\theta$

$4A\cos\theta + 4B\sin\theta - 4A\sin\theta + 4B\cos\theta = 6\sin\theta$

$(4A + 4B)\cos\theta + (-4A + 4B)\sin\theta = 6\sin\theta$

Comparing coefficients

$4A + 4B = 0$ - - - - (i)

$-4A + 4B = 6$ - - - - (ii)

Adding (1) and (2) $8B = 6$

$B = 3/4$

Substituting $4B = -4A$

$4(3/4) = 4A$

$3 = -4A$

$A = -3/4$

$\therefore y_p = -3/4 \cos\theta + 3/4 \sin\theta$

$y = y_h + y_p$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) + 3/4\sin\theta - 3/4\cos\theta$

iii) Steady state equation

At steady state

$y'_p = 0$

$y_p = 3/4\sin\theta - 3/4\cos\theta$

$y'_p = 3/4\cos\theta + 3/4\sin\theta = 0$

$\therefore 3/4\cos\theta = -3/4\sin\theta$

$\cos\theta = -\sin\theta$

Divide through by $\cos\theta$

$1 = -\tan\theta$

$\tan\theta = -1$

$\theta = \tan^{-1}(-1)$

$\theta = -45^{\circ}$

$\theta = 225^{\circ}$

$x_p = 1/10\cos t - 1/10\sin t = 0$

$dx_p/dt = 1/10(\cos t - \sin t) = 0$

2) $y = \sinh ax$

$y' = a\cosh ax$

$y'' = a^2\sinh ax$

$y''' = a^3\cosh ax$

$y^{iv} = a^4\sinh ax$

$y^n_{\sinh} = a^n/2[(1 + (-1)^n]\sinh ax + [1 - (-1)^n]\cosh ax]$

3) $y = \cosh ax$

$y' = a\sinh ax$

y $a^2\cosh ax$

2:) The equation : $EI\frac{d^2y}{dx^2} = \frac{W}{2}(l-x)^2$

$$y'' = \frac{w}{2EI}(l-x)^2$$

$$\frac{w}{2EI} = A_0$$

$$y'' = A_0 (l-x)^2$$

$$y'' = A(l^2-2xl+x^2)$$

$$y'' = Al^2 - 2Axl + Ax^2$$

assume $y''=0$

$$m^2 = 0$$

$$m = \sqrt{0}$$

$m = 0$ twice

$$\therefore y_p = e^{0x}(A+Bx)$$

$$y_p = A+Bx$$

Since $f(x) = Al^2-2Axl+Ax^2$, $y = Cx^4+Dx^3+Ex^2$

$$y' = 4Cx^3+3Dx^2+2Ex^2$$

$$y'' = 12Cx^2+6Dx+2E$$

$$\therefore 12Cx^2+6Dx+2E = Al^2-2AxL+Ax^2$$

$12C = A$ $\quad$ $6D = -2Al$ $\quad$ $2E = Al^2$

$12C = \frac{w}{2EI}$ $\quad$ $D = \cancel{x} -\frac{wl}{6EI}$ $\quad$ $2E = \frac{wl^2}{2EI}$

$$E = \frac{wl^2}{4EI}$$

$$\therefore y_p = \left(\frac{w}{24EI}\right)x^4 + \left(\frac{wl}{6EI}\right)x^3 + \left(\frac{wl^2}{4EI}\right)x^2$$

y general solution:

$$y = A+Bx+\left(\frac{w}{24EI}\right)x^4 - \left(\frac{wl}{6EI}\right)x^3 + \left(\frac{wl^2}{4EI}\right)x^2$$

$$y = A+Bx+\frac{w}{24EI}(x^4-4lx^3+6l^2x^2)$$

at $x=0$, $y=0$

$0 = A + 0 + 0$

$A = 0$

at $x = 0$, $y' = 0$

$$y' = B + \frac{w}{24EI}\left(4x^3 - 12Lx^2 + 12L^2x\right)$$

$$0 = B + \frac{w}{24EI}\left(0 - 0 + 0\right)$$

$0 = B$

Evaluate the value of y when $x = L$

$$y = \frac{w}{24EI}\left(L^4 - 4L^4 + 6L^4\right)$$

$$y = \frac{w}{24EI}\left(3L^4\right)$$

$$y = \frac{3L^4w}{24EI}$$

$$y = \frac{wL^4}{8EI}$$

Scanned by CamScanner