$$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0.$$

from eqn ①

$$x^2 y'' + xy' + y = 0.$$

For x^2y''

$$w^n = x^2y''$$

$$u = y'' \quad u^n = y^{n+2}$$

$$v = x^2 \quad v' = 2x \quad v'' = 2 \quad v''' = 0.$$

$$w^n = u^{(n)}v + nu^{(n-1)}v' + \frac{n(n-1)u^{(n-2)}}{2!}v'' + \frac{n(n-1)(n-2)}{3!}u^{(n-3)}v'''$$

$$= y^{n+2}\cdot x^2 + ny^{(n+1)}\cdot 2x + \frac{n(n-1)y^n\cdot \cancel{2}}{\cancel{2}\times 1} + 0$$

$$x^2y^{n+2} + 2xny^{(n+1)} + n(n-1)y^n$$

For xy'

$$u = y' \quad u^n = y^{n+1}$$

$$v = x \quad v' = 1 \quad v'' = 0$$

$$w^n = u^n v + nu^{(n-1)}v' + \frac{n(n-1)u^{(n-2)}}{2!}v''$$

$$w^n = y^{n+1}\cdot x + ny^n\cdot 1 + 0$$

$$w^n = xy^{n+1} + ny^n$$

For $w = y$

$$v = 1 \quad v' = 0$$

$$u = y \quad u^n = y^n$$

$$w^n = u^n v + nu^{(n-1)}v'$$

$$= y^n\cdot 1 + 0 = y^n$$

$$y^n = x^2y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n + xy^{(n+1)} + ny^n + y^n.$$

$$y = x^2y^{(n+2)} + 2xny^{(n+1)} + xy^{(n+1)} + n(n-1)y^n + ny^n + y^n.$$

$$x^2y^{(n+2)} + (xy + 2xn)y^{(n+1)} + [n(n-1) + n + 1]$$

$$x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + [n^2 - \cancel{n} + \cancel{n} + 1]y^n$$

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0.$$

$\rightarrow$ where $w^n = 2y$

$V = 2 \quad V' = 0$

$u = y \quad u^n = y^n.$

$W^n = u^n V^0$

$= y^n \cdot 2$

$= 2y^n.$

$\therefore y^n = y^{(n+2)} - [y^{(n+1)}(2x+1) + n2y^n] - [2y^n] = 0.$

$y^{(n+2)} = y^{(n+1)}(2x+1) + n2y^n + 2y^n$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2y^n(n+1).$

(2) $y = x^3 e^{4x}$ determine $y^{(5)}$

soln

$u = e^{4x} \qquad u^n = 4^n e^{4x}$

$V = x^3, \; V' = 3x^2, \; V'' = 6x, \; V''' = 6 \; V^{iv} = 0$

$$y^n = u^n \cdot V + nu^{(n-1)}V' + \frac{n(n-1)u^{(n-2)}V''}{2!} + \frac{n(n-1)(n-2)u^{(n-3)}V^3}{3!} + \frac{n(n-1)(n-2)(n-3)u^{(n-4)}V^4}{4!}$$

$$y^5 = 4^5 e^{4x} x^3 + 5 \cdot 4^{5-1} e^{4x} \cdot 3x^2 + \frac{5(5-1)4^{5-2}e^{4x} \cdot 6x}{2!} + \frac{5(5-1)(5-2)4^{(5}}{3!}$$

$$y^5 = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^4 e^{4x} 3x^2 + \frac{5 \cdot 4 \cdot 4^3 e^{4x} \cdot 6x}{2 \times 1} + \frac{5(4)(3) \cdot 4^2 e^{4x} \cdot 6}{3 \times 2 \times 1}$$

$$y^5 = 4^5 e^{4x} x^3 + 5 \cdot 4^4 e^{4x} 3x^2 + 5 \cdot 2 \cdot 4^3 e^{4x} 6x + 5 \cdot 4 \cdot 3 \cdot 4^2 e^{4x}$$

$$y^5 = 1024 e^{4x} x^3 + 3840x^2 e^{4x} + 3840xe^{4x} + 960e^{4x}$$

$$y^5 = 64e^{4x}[16x^3 + 60x^2 + 60x + 15]$$

(ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ —— ①

show that;

15/ENG06/002

Mechanical Engineering

Akindeni Suyenbarst

1 If $y = e^{x^2+x}$

$v = 1 \quad v' = 0$

$u = e^{x^2+x} \quad u^n = (2x+1)^n e^{x^2+x}$

$y = u^n v^0$

$y^n = (2x+1)^n e^{x^2+x} \cdot 1$

$y' = (2x+1)^1 e^{x^2+x}$

$v = 2x+1 \quad v' = 2 \quad v'' = 0$ (from y')

$u^n = (2x+1)^n e^{x^2+x}$

$y'' = u^n v^0 + n\, u^{(n-1)} v' + 0$

$= (2x+1)^1 e^{x^2+x} \cdot 2x+1 + n\,(2x+1)^{1-1} e^{x^2+x} \cdot 2 + 0$

$= (2x+1)^1 e^{x^2+x} (2x+1) + 1\,(2x+1)^0 e^{x^2+x} \cdot 2$

Recall $(2x+1)^1 e^{x^2+x} = y'$

$e^{x^2+x} = y$

$y'' = y'(2x+1) + 2y$

Prove that $y^{(n+2)} = (2x+1) y^{(n+1)} + 2(n+1) y^n$

where $w = y''$

from the above ans

$y'' - y'(2x+1) - 2y = 0$

$v = 1 \quad v' = 0$

$u = y'' \quad u^n = y^{(n+2)}$

$w^n = u^n v + 0$

$w^n = y^{(n+2)} \cdot 1$

$w^n = y^{(n+2)}$

Where $w^n = (2x+1) y$

$v = 2x+1 \quad v' = 2 \quad v'' = 0$

$u = y' \quad u^n = y^{(n+1)}$

$w^n = u^n \cdot v^0 + n u^{(n-1)} v'$

$w^n = y^{(n+1)} (2x+1) + n y^n \cdot 2$