AASTRUP OLUWADOYINSOLA MERCY

S/ENG06/032

MECHANICAL ENGINEERING

ENG 381

ASSIGNMENT #3

$y = e^{x^2+x}$

$v = 1 \quad v' = 0$

$u = e^{x^2+x} \quad u^n = (2x+1)^n e^{x^2+x}$

$y = u^n \cdot v^0$

$y^n = (2x+1)^n e^{x^2+x} \cdot 1$

$y' = (2x+1)' e^{x^2+x}$

$v = 2x+1, \; v' = 2 \quad v'' = 0$ (from y')

$y^n = (2x+1)^n e^{x^2+x}$

$y'' = u^n v^0 + n u^{(n-1)} v' + 0$

$= (2x+1)' e^{x^2+x} \cdot 2x+1 + n(2x+1)^{1-1} e^{x^2+x} \cdot 2 + 0$

$= (2x+1)' e^{x^2+x} \cdot (2x+1) + 1(2x+1)^0 e^{x^2+x} \cdot 2$

Recall $(2x+1)' e^{x^2+x} = y'$

$e^{x^2+x} = y$

$y'' = y'(2x+1) + 2y$

Proof $y^{(n+2)} = (2x+1) y^{(n+1)} + 2(n+1) y^n$

where $w = y''$

from the above;

$y'' - y'(2x+1) - 2y = 0$

$v = 1 \quad v' = 0$

$u = y'' \quad u^n = y^{(n+2)}$

$\rightarrow \quad w^n = v^n \cdot u + 0$

$w^n = y^{(n+2)} \cdot 1$

$w^n = y^{(n+2)}$

$\rightarrow$ when $w^n = (2x+1)y$

$v = 2x+1 \quad v' = 2 \quad v'' = 0$

$u = y' \quad u^n = y^{(n+1)}$

$W^n = u^n v^n + n u^{(n-1)} v'$

$w^n = y^{(n+1)} (2x+1) + n y^n \cdot 2$

where $w^n = 2y$

$v = 2 \quad v' = 0$

$u = y \quad u^n = y^n$

$W^n = u^n v^0$

$= y^n \cdot 2$

$= 2y^n$

$\therefore y^n = y^{(n+2)} - [y^{(n+1)}(2x+1) + n2y^n] - [2y^n] = 0$

$y^{(n+2)} = y^{(n+1)}(2x+1) + n2y^n + 2y^n$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2y^n(n+1)$

$y = x^3 e^{4x}$ determine $y^{(5)}$

Soln

$y = e^{4x} \quad \cdots \quad y^n = 4^n e^{4x}$

$v = x^3, \quad v' = 3x^2, \quad v'' = 6x, \quad v''' = 6 \quad v^{iv} = 0$

$$y^n = u^n \cdot v + n u^{(n-1)} v' + \frac{n(n-1)u^{(n-2)} v^2}{2!} + \frac{n(n-1)(n-2)u^{(n-3)} v^3}{3!}$$

$$\frac{n(n-1)(n-2)(n-3)u^{(n-4)} v^4}{4!}$$

$y^5 = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^{5-1} e^{4x} \cdot 3x^2 + \frac{5(5-1) 4^{5-2} e^{4x} \cdot 6x}{2!} + \frac{5(5-1)(5-2) 4^{(5-3)} e^{4x} 6}{3!}$

$y^5 = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^4 e^{4x} 3x^2 + \frac{5 \cdot 4 \cdot 4^3 e^{4x} \cdot 6x}{2 \times 1} + \frac{5(4)(3) \cdot 4^2 e^{4x} 6}{3 \times 2 \times 1}$

$y^5 = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^4 e^{4x} 3x^2 + 5 \cdot 2 \cdot 4^3 e^{4x} 6x + 5 \cdot 4 \cdot 5 \cdot 4^2 e^{4}x$

$y^5 = 1024 e^{4x} x^3 + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$

$y^5 = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15)$

(ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ - - - (1)

Show that:

$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0$

from eqn (1)

$x^2 y'' + x y' + y = 0$

for $x^2 y''$

Let $x^2 y''$

$u = y''$ $\quad u^n = y^{n+2}$

$v = x^2$ $\quad v' = 2x$ $\quad v'' = 2$ $\quad v''' = 0$

$w^n = u^{(n)} v^0 + n u^{(n-1)} v' + \frac{n(n-1) u^{(n-2)} v''}{2!} + \frac{n(n-1)(n-2) u^{(n-3)} v'''}{3!}$

$= y^{n+2} \cdot x^2 + n y^{(n+1)} \cdot 2x + \frac{n(n-1) y^n \cdot 2}{2 \times 1} + 0$

$= x^2 y^{n+2} + 2xn y^{(n+1)} + n(n-1) y^n$

for xy'

$u = y'$ $\quad u^n = y^{n+1}$

$v = x \quad v' = 1 \quad v'' = 0$

$$w^n = y^n v + n y^{(n-1)} v' + \frac{n(n-1) y^{(n-2)} v''}{2!}$$

$$w^n = y^{n+1} \cdot x + n y^n \cdot 1 + 0$$

$$w^n = x y^{n+1} + n y^n$$

For $w = y$

$v = 1 \quad v' = 0$

$u = y \quad u^n = y^n$

$$w^n = u^n v + n u^{(n-1)} v'$$

$$= y^n \cdot 1 + 0 = y^n$$

$$y^n = x^2 y^{(n+2)} + 2xn y^{(n+1)} + n(n-1) y^n + x y^{(n+1)} + n y^n + y^n$$

$$y = x^2 y^{(n+2)} + 2xn y^{(n+1)} + x y^{(n+1)} + n(n-1) y^n + n y^n + y^n$$

$$x^2 y^{(n+2)} + (xy + 2xn) y^{(n+1)} + (n - (n-1) + n + 1]$$

$$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0$$

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