lb/Eng 04/009

$\lim \left[\frac{\pi}{2} - \frac{\pi}{4} \right) \sin(\cos x)$

$$\frac{\left[x^2 - \frac{\pi}{4} \right) \sin(\cos x)}{\frac{\pi}{2} - \frac{\pi}{2}} \to 0 \text{ undefined}$$

$$\lim \frac{\frac{d}{x}\left(x^2 - \frac{\pi}{4} \right) \sin(\cos x) \big]}{x - \frac{\pi}{2}}$$

for numerator

$y = x^2 = \pi/4 \sin(\cos x)$

$y = x^2 - \pi/4$

$v = \sin(\cos x)$

let $= \cos x \quad v - \sin u$

$\frac{dy}{dx} = dv/du \times du/dv$

$\cos u \times \sin u$

$u = \cos v$

$= -\sin x (\cos x) \cos$

$y = uv$

$dy/dx = u\, dv/dx + v\, du/dx$

$(x^2 - \pi/4) - \sin x \cos(\cos x) + \sin(\cos(x)) 2x$

$\frac{dy}{dx} = -(x^2 - \pi/4) \sin x \cos(\cos x + 2 \sin(\cos x))$

for denominator

$y = x - \pi/2 \quad dy/dx = 1$

$$= \frac{(x^2 - \pi/4 \ \sin\cos(\cos x + 2\sin(\cos x))}{1}$$

$(\pi/2)^2 - \pi/4 \sin\cos(\cos \pi/2)$

$\pi \; 180°$

x/y $\pi/4 \sin\cos(\cos 90)$ $2[\pi/2]\sin\cos 0 90)$

$(\pi/2)^2 - \frac{x}{2} \sin 90 \cos 0 + 2\frac{\pi}{2} \sin 0$

$(\frac{\pi}{2})^2 - \frac{x}{4} \sin 90 \; 1 + 2(\frac{\pi}{2})$

$-(\frac{\pi}{2})^2 - \frac{x}{4} \; (1) + 0$

$\frac{dy}{dx} = - \; (\frac{\pi}{2})^2 - \frac{\pi}{4}$

$\lim_{x \to \pi/2} \ln\left(\frac{\text{Exp}(3x^2+2x+1)}{x+1}\right)$

factoring the numerator

$\frac{(3x-1) \quad (x+1)}{x+1}$

$3(\frac{\pi}{2}) - 1$

$\frac{3\pi}{2} - 1$

$\lim_{x \to 2+\sqrt{3}} \cos\left(\sin\left(\frac{x-2}{(x-\sqrt{3})}\right)\right)$

$\cos\left[x + n^{-1}\left[\frac{2 \; \sqrt{3} - x}{2 + \sqrt{3} - \sqrt{3}}\right)\right.$

$\cos\{\sin^{-1}(\frac{\sqrt{3}}{2}))$

$\cos(0.4660)$

$\cos 60 = \frac{1}{2}$

$\lim_{x \to 2+\sqrt{3}} \cos\left(\sin^{-1}\frac{x-2}{(x-\sqrt{3})}\right) = \frac{1}{2}$

$\lim_{x \to 4} \left(\frac{x^2-8x+16}{x^2-5+4}\right) = \ldots 0$

$\lim_{x \to 4} \left(\frac{2x-8}{2x-5}\right.$

$\frac{8-8}{8-5} = \frac{0}{3} = 0$

2a

$\frac{2}{2\times 3} + \frac{2}{3\times 4} + \frac{2}{4\times 5} + \frac{2}{5\times 6}$

$u_n = \frac{2}{(n+1)(n+2)}$ $\quad u_{n+1} = \frac{2}{(n+1)+1)((n+1)+2)}$

$u_{n+1} = \frac{2}{(n+2)(n+3)}$

using D Albert

$\frac{u_{n+1}}{u_n} = \frac{2}{(\cancel{n+2})(n+3)} \times \frac{(\cancel{n+1})(n+2)}{2}$

$\frac{u_{n+1}}{u_n} = \frac{n+1}{n+3}$

$\lim_{n\to\infty} \left(\frac{n+1}{n+3}\right) = \left(\frac{n/n + 1/n}{n/n + 3/n}\right)$ $\quad 1/n \to 0$ $\quad 3/n \to 0$

$\frac{1+0}{1+0} = \frac{1}{1} = 1$

Therefore is ~~it~~ it inconclusive

2b

$\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2}$

$u_n = \frac{2}{n^2}$

$u_{n+1} = \frac{2}{(n+1)^2}$

$\frac{u_{n+1}}{u_n} = \frac{2}{(n+1)^2} \times \frac{n^2}{2}$

$\lim_{n\to\infty} \left(\frac{u_{n+1}}{u_n}\right) = \frac{n^2}{(n+1)^2} = \frac{n^2}{n^2+2n+1)}$

$\frac{u_{n+1}}{u_n} = \frac{n^2}{n^2} = \frac{1}{1+\frac{2}{n}+\frac{1}{n^2}}$ $\quad = \frac{2}{n} \to 0$

$\frac{1}{n} \to 0$

$\frac{u_{n+1}}{u_n} = \frac{1}{1+0+0} = 1$

using comparison test

$\frac{2}{n^2}$ $\frac{1}{n}$

$\frac{2}{1^2} < \frac{1}{1^2}$

Therefore it's converging

2c

$\frac{1n^{-1}+2n^2}{1+n^2}$ since the n^{th} term is given (u_n)

$$u_n = \frac{\frac{1}{n^2} + \frac{2n^3}{n^2}}{\frac{1}{n} + \frac{n^2}{n^2}}$$

$\frac{1}{n^2} = 0$ $\frac{1}{n^2} \to 0$ / $\frac{0+2}{0+1} = \frac{2}{1}$

3) $\frac{x}{27} + \frac{x^2}{12} + \cdots + \frac{x^n}{(2n+1)^3}$

$$u_n = \frac{n^x}{(2n+1)^3}$$

$$u_{n+1} = \frac{x^{n+1}}{(2(n+1)+1)^3}$$

$$u_{n+1} = \frac{n+1}{(2n+2)^3}$$

$$u_{n+1} = \frac{n^{n+1}}{2n+2^3} \times \frac{2(n+1)^3}{n^2}$$

$$\frac{x^2-n}{2(n+2)^2} \times \frac{2(n+1)^3}{n^x}$$

$$x \frac{(n+1)^3}{(n+2)^3}$$

expanding the bracket into have

$$x\left(\frac{n^3+3n^2+3n+1}{n^3+6n^2+12n+8}\right)$$

$$x\left(\frac{1+\frac{3}{n}+\frac{3}{2n}+\frac{1}{n^3}}{1+\frac{6}{n}+\frac{12}{n^2}+\frac{8}{n^3}}\right)$$

$$= x\left(\frac{1+0+0+0}{1+0+0+0}\right) \quad x\left(\frac{1}{1}\right) = x$$