EFENDU DIVINE

15/ENGE6/026

MECHANICAL ENGINEERING

ASSIGNMENT

1 $d^2y/dx^2 - dy/dx - 2y = 8$

Recall

For non-homogeneous equation

General solution = complementary function (cf) + Particular Integral (P.I)

Complementary function (cf)

$d^2y/dx^2 - dy/dx - 2y = 0$

The auxiliary equation becomes

$m^2 - m - 2 = 0$

$m^2 - 2m + m - 2 = 0$

$m(m-2) + 1(m-2) = 0$

$m + 1 = 0$ or $m - 2 = 0$

$m = -1$ $m = 2$ ⇒ real different roots

PI

$y = c$ --- ①

$dy/dx = 0$ --- ②

$d^2y/dx^2 = 0$ --- ③

Substitute equ ①, ② and ③ into general equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$$

$0 - 0 - 2(c) = 8$

$-2c = 8$

$c = \frac{8}{-2} \Rightarrow c = -4$

$y = c$

$y = -4$

General solution

CF + PI

$\therefore y = Ae^{-x} + Be^{2x} + (-4)$

$y = Ae^{-x} + Be^{2x} - 4$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

Find the CF

$CF = \frac{d^2y}{dx^2} - 4y = 0$

auxiliary equation becomes

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm\sqrt{4}$

$m = \pm 2$

$m = 2$ or $m = -2$

$CF = Ae^{2x} + Be^{-2x}$

PI, Assume PI

$y = Ce^{3x}$ --- (1)

$\frac{dy}{dx} = 3Ce^{3x}$ --- (2)

$\frac{d^2y}{dx^2} = 9Ce^{3x}$ --- (3)

Substitute (1), (2) and (3) into the general equation

$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$e^{3x}(9C - 4C) = 10e^{3x}$

$9C - 4C = 10$

$5C = 10$

$C = 10/5$

$C = 2$

$y = Ce^{3x}$

$y = 2e^{3x}$

General solution

$y = Ae^{2x} + Be^{-2x} + 2e^{3x}$

3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

auxiliary equation becomes

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$m = -1$ or $m = -1$

$CF = e^{-x}(A + Bx)$

PI, Assume PI

$y = Ce^{-2x}$ --- (1)

$\frac{dy}{dx} = -2Ce^{-2x}$ --- (2)

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$ --- (3)

$Ce^{-2x} = e^{-2x}$

$C = 1$

$y = Ce^{-2x}$

$y = e^{-2x}$

General solution

$CF + PI$

$y = e^{-x}(A + Bx) + e^{-x}$

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

Auxiliary equation becomes

$\frac{d^2y}{dx^2} + 25y = 0$

$m^2 + 25 = 0$

$m^2 = -25$

$m = \pm\sqrt{-25} \Rightarrow m = \pm\sqrt{25} \cdot \sqrt{-1}$

$m = \pm 5i$ (complex roots)

Cf. comparing with $m = \alpha \pm \beta j$

$\alpha = 0, \beta = 5$

$CF = e^{0x}(A\cos 5x + B\sin 5x)$

Since $e^0 = 1$

PI, Assume PI

$y = Cx^2 + Dx + E$ --- ①

$\frac{dy}{dx} = 2C + D$ --- ②

$\frac{d^2y}{dx^2} = 2C$ --- ③

Substitute equ ①, ② and ③ into general equation

$\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x + 2C + 25Cx^2$

$+ 25Dx + 25E = 5x^2 + x$

$25Cx^2 = 5x$

$25C = 5$

$C = 5/25$

$C = 1/5$ --- ④

$25Dx = x$

$25D = 1$

$D = 1/25$ --- ⑤

$2C + 25E = 0$

recall $C = 1/5$

$2(1/5) + 25E = 0$

$2/5 + 25E = 0$

$25E = 2/5$

$E = 2/5 \times 1/25$

$E = 2/125$ --- ⑥

Substitute equ ④ ⑤ and ⑥ into ①

$y = Cx^2 + Dx + E$

$y = 1/5\,x^2 + 1/25\,x + 2/125$

general solution

$CF + PI$

$y = A\cos 5x + B\sin 5x + (1/5\,x^2 + 1/25\,x + 2/125)$

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$CF = \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

auxillary equation becomes

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$m^2 = 1$ (twice)

$CF = e^x(A + Bx)$

Particular integral

Assume PI

$y = C\cos x + D\sin x$ --- ①

$\frac{dy}{dx} = -C\sin x + D\cos x$ --- ②

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$ --- ③

Substitute ①, ② and ③ into original equation

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x$

$x = 4\sin x$

$= 2C\sin x \cdot 2D\cos x = 4\sin x$

comparing the coefficient

$2C = 4$

$-2D = 0$

$C = 4/2$ --- (4)

$D = 0$ --- (5)

$y = C\cos x + D\sin x$

$y = 2\cos x + 0\sin x$

$y = 2\cos x$ --- (6)

general solution

$CF + PI$

$y = e^{x}(A + Bx) + 2\cos x$

6 $\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 5 = 2e^{-2x}$

given that $x = 0$, $y = 1$

$dy/dx = -2$

non-homogeneous equation

$GS = CF + PI$

$= d^2y/dx^2 + 4\,dy/dx + 5y = 0$

auxiliary equation becomes

$m^2 + 4m + 5 = 0$

using quadratic formula

$a = 1, b = 4, c = 4$

$\frac{-4 \pm \sqrt{4^2 - (4 \times 1 \times 5)}}{2 \times 1}$

$= \frac{-4 \pm \sqrt{16 - 20}}{2}$

$\sqrt{-4} = \sqrt{4} \times \sqrt{-1} = 2j$

$\frac{-4 \pm 2j}{2} = -2 \pm j$

comparing with $m = \alpha + \beta j$, $\alpha = -2$, $\beta = 1$

$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

$y = e^{-2x}(A\cos x + B\sin x)$

PI, Assume PI

$y = Ce^{-2x}$

But since Ce^{-2x} is same to CF and PI, multiply through by x $y = xCe^{-2x}$

$y = Cxe^{-2x}$ --- (1)

$dy/dx = -2Cxe^{-2x}$ --- (2)

$d^2y/dx^2 = 4Cxe^{-2x}$ --- (3)

Substitute equ (1) (2) and (3) into the general equation

$d^2y/dx^2 + 4\,dy/dx + 5y = 2e^{-2x}$

$4Cxe^{-2x} + 4(-2Cxe^{-2x}) + 5Cxe^{-2x} = 2Ce^{-2x}$ (4)

$4Cxe^{-2x} - 8Cxe^{-2x} + 5Cxe^{-2x} = 2Ce^{-2x}$ --- (5)

$Cxe^{-2x}(1) = 2Ce^{-2x}$

$Cx = 2$

$C = 2/x$ --- (6)

Substitute equ (6) into (1)

$y = Cxe^{-2x}$

$y = 2/x \times e^{-2x}$

The general solution becomes

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

values for A and B when $x = 0$, $y = 0$

$1 = e^{-2(0)}(A\cos(0) + B\sin(0)) \cdot 2e^{-2(0)}$

$1 = 1(A \cdot 1 + (B \cdot 0)) + 2$

$1 = A + 2$

$A = 1 - 2$

$A = -1$

when $x = 0$ $dy/dx = -2$

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

$y = e^{-2x}A\cos x + e^{-2x}B\sin x + 2e^{-2x}$

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Using product rule

$dy/dx = e^{-2x}(\sin Ax) - (2e^{-2x})$

$dy/dx = -e^{2x}(A\sin x) - 2e^{-2x}(A\cos x) + e^{-2x}$
$(B\cos x) - 2e^{-2x}(B\sin x) - 4e^{-2x}$

$= -2 = e^{-2(0)}(A\sin(0)) - 2e^{-2(0)}(A\cos(0))$
$+ e^{-2(0)}(B\cos(0)) - 2e^{-2(0)}(B\sin(0))$
$- 2e^{-2(0)}(B\sin(0)) - 4e^{-2(0)}$

$-2 = 0 - 2(A) + B - 0 - 4$

$-2 = -2A + B - 4$

$-2A + B = -2 + 4$

recall $A = -1$

$-2(-1) + B = 2$

$2 + B = 2$

$B = 0$

Substitute $A = -1$ and $B = 0$ into the equation

$y = e^{-2x}(-1\cos x + (0)\sin x + 2e^{-2x}$

$y = e^{-2x}(-\cos x) + 2e^{-2x}$

$y = e^{-2x}(2 - \cos x)$

7 $3\,d^2y/dx^2 - 2\,dy/dx - y = 2x - 3$

auxiliary equation becomes

$3m^2 - 2m - 1 = 0$

$m = 2$ or $m = 1/3$ recall; different root

$Cf = Ae^{x} + Be^{-1/3x}$

PI, Assume PI

$y = Cx + D$ ---- ①

$dy/dx = C$ --- ②

$d^2y/dx = 0$ --- ③

substitute ①, ② and ③ into the general equation

$3(0) - 2(C) - (Cx + D) = 2x - 3$

$0 - 2C - (x - D) = 2x - 3$

comparing coefficient

$-Cx = 2x$

$-C = 2$

$C = 2$ --- ④

$-2C - D = -3$

recall $C = -2$

$-2(-2) - D = 3$

$4 - D = 3$

$D = 4 - 3$

$D = 1$ ---- ⑤

$y = -2x + 1$

general solution becomes

$y = Ae^{x} + B^{-1/3x} - 2x + 1$

8) $\dfrac{d^2y}{dx^2} + \dfrac{6dy}{dx} + 8y = 8e^{4x}$

non-homogeneous equation

$CF = \dfrac{d^2y}{dx^2} - \dfrac{6dy}{dx} + 8y = 0$

auxiliary equation is

$m^2 - 6m + 8 = 0$

$m^2 - 4m - 2m + 8 = 0$

$m(m-4) - 2(m-4) = 0$

$m = 4$ or $m = 2$ recall; different roots

$CF = Ae^{4x} + Be^{2x}$

PI, Assume PI

$y = Ce^{4x}$ --- ①

CF and PI are the same

multiply by x

$y = Cxe^{4x}$ --- ②

product rule

$= C\left(x16e^{4x} + 4e^{4x} + 4e^{4x}\right)$

$= C\left(x16e^{4x} + 8e^{4x}\right)$ ---- ③

Substitute eqn ①, ② and ③ into the general equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$$

$$(x16e^{4x} + (8e^{4x} - 6C(x4e^{4x} + Ce^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$$

$$Cx16e^{4x} + C8e^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$$

$$Cxe^{4x}(16 - 24 + 8) + Ce^{4x}(8 - 6) = 8e^{4x}$$

$$0 + Ce^{4x}(2) = 8e^{4x}$$

$$2C = 8$$

$$C = \frac{8}{2}$$

$$C = 4$$

$$y = Cxe^{4x}$$

$$y = 4xe^{4x}$$

$$GS = CF + PI$$

$$y = Ae^{4x} + Be^{2x} + 4xe^{4x}$$