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15/ENG06/043
Mechanical Engineering
ENG 381

(1) $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

$\text{Cot } 6\sin\theta = 0$

$m^2 + 4m + 5 = 0$

$a = 1, \ b = 4, \ c = 5$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{4^2 + 4 \times 1 \times 5}}{2 \times 1}$

$= \frac{-4 \pm j\sqrt{-4}}{2}$

$= \frac{-4 \pm j2}{2}$

$m = -2 \pm j$

$m = \alpha \pm j\beta$

$C.f = y = e^{-2x}(A\cos\theta + B\sin\theta)$

$f(x) = 6\sin\theta$

$P.I = y = (C\cos\theta + D\sin\theta)$

$\frac{dy}{d\theta} = -C\sin\theta + D\cos\theta$

$\frac{d^2y}{d\theta^2} = -C\cos\theta - D\sin\theta$

$\frac{d^2y}{d\theta^2} + \frac{4dy}{d\theta} + 5y = 6\sin\theta$

$-(C\cos\theta - D\sin\theta + 4(-C\sin\theta + D\cos\theta) + 5C\cos\theta + 5D\sin\theta$

$= 6\sin\theta$

$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$

$4C\cos\theta + 4D\sin\theta - 4\sin\theta + 4D\cos\theta = 6\sin\theta$

$4C\cos\theta + 4D\sin\theta + 4D\sin\theta - 4C\sin\theta = 6\sin\theta$

$(4C+4D)\cos\theta + (4D-4C)\sin\theta = 6\sin\theta$

$4C + 4D = 0 \quad \text{---} (1)$

$-4C + 4D = 6 \quad \text{---} (2)$

$8D = 6$

$D = \frac{3}{4}$

from eqn (1)

$4C + 4\left(\frac{3}{4}\right) = 0$

$4C + 3 = 0$

$4C = -3$

$C = -\frac{3}{4}$

$P.I = y = \frac{-3}{4}\cos\theta + \frac{3}{4}\sin\theta$

$G.S = C.F + P.I$

$y = e^{-2x}(A\cos\theta + B\sin\theta) - \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

Steady state equation

$y' = \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta = 0$

$\frac{3/4\sin\theta}{3/4\cos\theta} = \frac{3/4\cos\theta}{3/4\cos\theta}$

$\tan\theta = -1$

$\theta = \tan^{-1}(-1)$

$\theta = -45°$

2. $EI\frac{d^2y}{dx^2} = \frac{w(L-x)^2}{2}$

// Convert Equation into an homogeneous equation

$EI\frac{d^2y}{dx^2} = 0$

$EIm^2 = 0$

$m^2 = 0 \Rightarrow m = \sqrt{0} = 0 \quad m_1 = m_2 = 0$

$y = e^{0x}(A + Bx)$

$y = A + Bx$ // CF

$y = Ax^2 + Sx^3 + Tx^4$

$\frac{dy}{dx} = 2Rx + 3Sx^2 + 4Tx^3$

$\frac{d^2y}{dx^2} = 2R + 6Sx + 12Tx^2$

$EI[2R + 6Sx + 12Tx^2] = \frac{w}{2}(L-x)^2$

$2REI + 6SxEI + 12Tx^2EI = \frac{w}{2}[L^2 - 2Lx + x^2]$

// multiply eqn by 2

$4REI + 12SxEI + 24Tx^2EI = wL^2 - 2wLx + wx^2$

$24TEI = w$

$T = \frac{w}{24EI}$

$12SEI = -2wL$

$S = \frac{-2wL}{24EI}$

$4REI = wL^2$

$R = \frac{wL^2}{4EI}$

$y = \left[\frac{wL^2}{4EI}\right]x^2 - \left[\frac{wL}{6EI}\right]x^3 + \left[\frac{w}{24EI}\right]x^4$

$y = \frac{wL^2x^2}{4EI} - \frac{wx^3}{6EI} + \frac{wx^4}{24EI}$

$$y = \frac{6wL^2x^2 - 4wLx^3 + wx^4}{24EI}$$

$$y = \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

$$y = A + Bx + \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

at $x = 0$, $y = 0$ $\quad \frac{dy}{dx} = 0$

$$0 = A + B(0) + \frac{w}{24EI}\left[6L^2(0)^2 - 4L(0)^3 + (0)^4\right]$$

$$A = 0$$

$$\frac{dy}{dx} = B + \frac{w}{24EI}\left[12L^2x^2 - 12Lx^2 + 4x^3\right]$$

$$0 = B + \frac{w}{24EI}\left[12L^2(0) - 12L(0)^2 + 4(0)^3\right.$$

$$B = 0$$

when $A = B = 0$

$$y = 0 + 0x + \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

$$y = \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

when $x = L$

$$y = \frac{w}{24EI}\left[6L^4 - 4L^4 + L^4\right.$$

$$y = \frac{w}{24EI}\left[3L^4\right]$$

$$y = \frac{wL^4}{8EI}$$