ELIJAH DIVINE
15/ENG/0126
MECHANICAL ENGINEERING
ENG 381

1. $d^2y/dt^2 + 4dy/dt + 5y = 6\sin t$

reduced equation to homogeneous equation

$d^2y/dt^2 + 4dy/dt + 5y = 0$

$m^2 + 4m + 5 = 0$

$a = 1, \quad b = 4, \quad c = 5$

$m = \dfrac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1}$

$m = \dfrac{-4 \pm j\sqrt{4}}{2}$

$m = -4 \pm j^{4/2}$

$m = -2 \pm j$

complementary function

$y = e^{-2t}(A\cos t + B\sin t)$

$y = C\cos t + D\sin t$

$dy/dt = -(C\sin t + D\cos t)$

$d^2y/dt^2 = -C\cos t + D\sin t$

Substitute y, dy/dt, d^2y/dt^2 into the equation

$-C\cos t - D\sin t + 4(-C\sin t + D\cos t) + 5(C\cos t + D\sin t) = 6\sin t$

$-C\cos t - D\sin t = 4C\sin t + 4D\cos t + 5C\cos t + 5D\sin t = 6\sin t$

$-C + 5C = 0$

$4D + 4C = 0 \quad \cdots (1)$

$-D + 4C + 5D = 6$

$4D + 4C = 6 \quad \cdots (2)$

equating both equations

$4D + 4C = 0 \quad \cdots (1)$

$4D + 4C = 6 \quad \cdots (2)$

$8C = 6$

$C = 3/4$

Substitute c into equation (2)

$4D + 4(3/4) = 6$

$4D + 3 = 6$

$4D = 6 - 3$

$D = 3/4$

Particular integral

$y = 3/4 \cos\theta + 3/4 \sin\theta$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) + 3/4\cos\theta + 3/4\sin\theta$

GS = CF + PI

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) + 3/4(\cos\theta + \sin\theta)$

at steady state

$\theta = \infty \quad dy/d\theta = 0$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) + 3/4(\cos\theta + \sin\theta)$

$dy/d\theta = -2e^{-2\theta}(-A\sin\theta + B\cos\theta) + 3/4[-\sin\theta + \cos\theta]$

$0 = -2e^{-2\infty}(B\cos\infty - A\sin\infty) + 3/4(\cos\infty - \sin\infty)$

$3\cos\infty + 3\sin\infty = 0$

$0 = 3/4\cos\infty - 3/4\sin\infty$

$\frac{3/4\sin\infty}{3/4\cos\infty} = \frac{3/4\cos\infty}{3/4\cos\infty}$

$\tan\infty = 1$

$\infty = \tan^{-1} 1$

2 $\frac{1}{2} d^2y/dx^2 = c/2(1-x)^2$

convert equation to homogenous equation

$1 \cdot 1 \; d^2y/dx^2 = 0$

$1 \cdot 1 \; m^2 = 0$

$m^2 = 0$

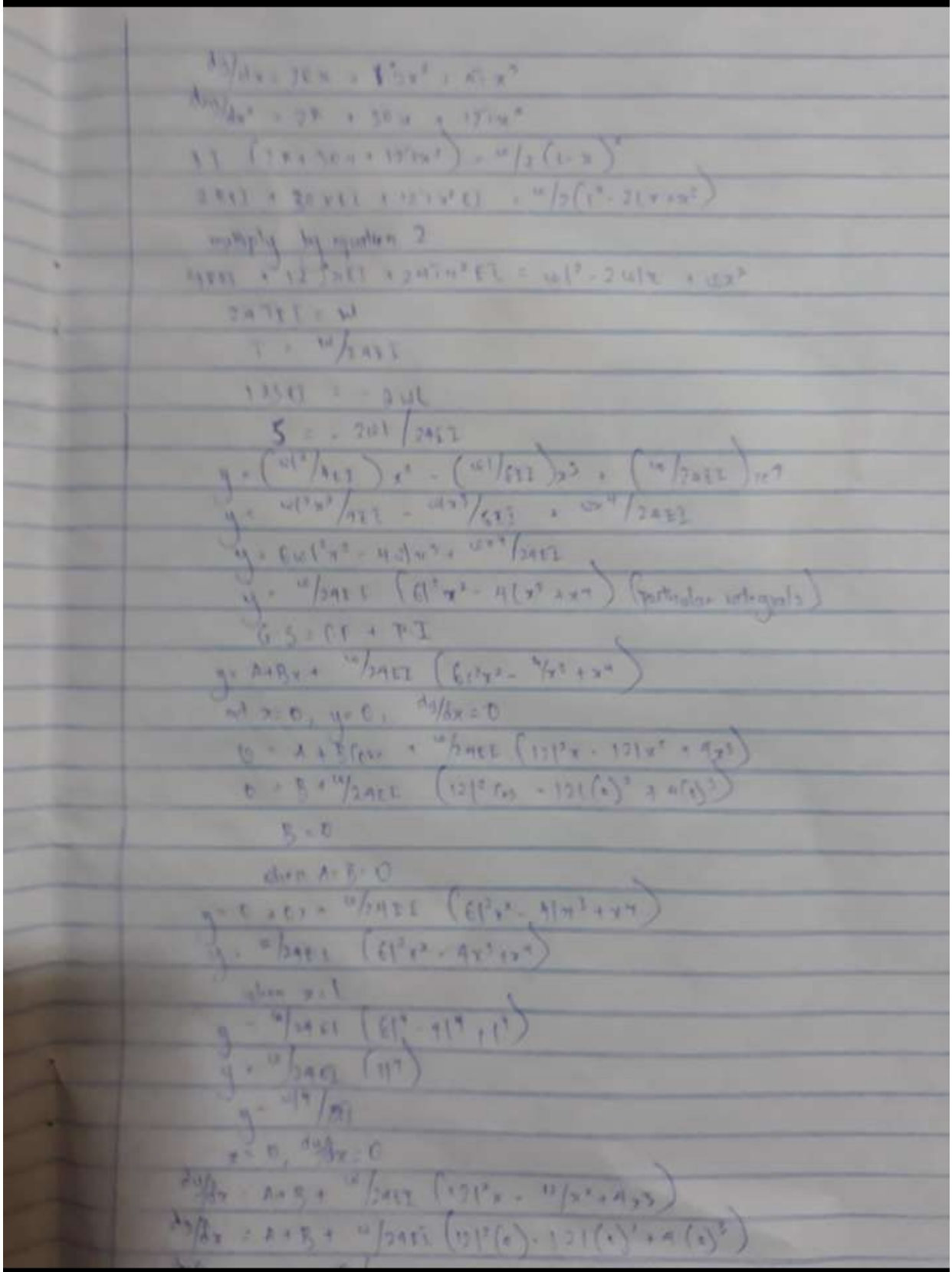
$m = \pm\sqrt{0}$

$m_1 = m_2 = 0$

$y = e^{0x}(A + Bx)$

complementary portion

$y = A + Bx$



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