$-2C - D = 3$

recall $C = -2$

$-2(-2) - D = 3$

$4 - D = 3$

$D = 4 - 3$

$D = 1$ - (5)

$y = -2x + 1$

general solution become

$y = Ae^{x} + B^{-1/3x} - 2x + 1$

(8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

non-homogenous equation

$CF = \frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

auxilary equation is

$m^2 - 6m + 8 = 0$

$m^2 - 4m - 2m + 8 = 0$

$m(m-4) - 2(m-4) = 0$

$m = 4$ & $m = 2$ (reals Different roots)

$CF = Ae^{4x} + Be^{2x}$

PI

$= y = Ce^{4x}$ - (1)

cf and PI are the same

~~divide by x~~

multiply by x

$y = Cxe^{4x}$ - (1)

Product rule

$= C(x16e^{4x} + 4e^{4x} + 4e^{4x})$

$= C(x16e^{4x} + 8e^{4x})$ - (3)

Sub (1), (2), (3) into the general equation

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$Cx16e^{4x} + C8e^{4x} - 6(Cx4e^{4x} + Ce^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$

$Cx16e^{4x} + C8e^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$

$Cxe^{4x}(16 - 24 + 8) + Ce^{4x}(8 - 6) = 8e^{4x}$

$0 + Ce^{4x}(2) = 8e^{4x}$

$2C = 8$

$C = 8/2$

$C = 4$

$y = Cxe^{4x}$

$y = 4xe^{4x}$

$GS = CF + PI$

$y = Ae^{4x} + Be^{2x} + 4xe^{4x}$

$1 = e^{-2(0)}(A\cos(0) + B\sin(0)) + 2e^{-2(0)}$

$1 = 1(A \cdot 1 + (B \cdot 0) + 2$

$1 = A + 2$

$A = 1 - 2$

$A = -1$

When $x = 0$ $dy/dx = -2$

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

$y = e^{-2x}A\cos x + e^{-2x}B\sin x + 2e^{-2x}$

using product rule

$dy/dx = -e^{-2x}[\sin Ax] - 2e^{-2x}$

~~ut~~

$dy/dx = -e^{2x}[A\sin x) - 2e^{-2x}[A\cos x] + e^{-2x}(B\cos x) - 2e^{-2x}[B\sin x] - 4e^{-2x}$

$= -2 = e^{-2(0)}[A\sin(0)] - 2e^{-2(0)}[A\cos(0)] + e^{-2(0)}[B\cos(0)] - 2e^{-2(0)}[B\sin(0)] - 4e^{-2(0)}$

$-2 = 0 - 2(A) + B - 0 - 4$

$-2 = -2A + B - 4$

$-2A + B = -2 + 4$

recall $A = -1$

$-2(-1) + B = 2$

$2 + B = 2$

$B = 2 - 2$

sub $A = -1$ & $B = 0$ into the equation

$y = e^{-2x}(-1\cos x + 0\sin x) + 2e^{-2x}$

$y = e^{-2x}(-\cos x) + 2e^{-2x}$

$y = e^{-2x}(2 - \cos x)$,

(2) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

Auxilary equation becomes

$3m^2 - 2m - 1 = 0$

$m = 1$ & $m = -1/3$ reals different root

$cf = Ae^{x} + Be^{-1/3x}$

Particular integral

Assumed P.I

$y = Cx + D$ - (1)

$\frac{dy}{dx} = C$ - (2)

$\frac{d^2y}{dx^2} = 0$ — (3)

sub eqn (1), (2), (3) in the general equation

$3(0) - 2(C) - (Cx + D) = 2x - 3$

$0 - 2C - Cx - D = 2x - 3$

Comparing coefficients

$-Cx = 2x$

$-C = 2$

$C = -2$ -- (4)

$$\frac{\partial^2 y}{\partial x^2} + 4\frac{\partial y}{\partial x} + 5 = 2e^{-2x}$$

given at $x = 0, y = 1$

& $\frac{\partial y}{\partial x} = -2$

non homogenous equation

$GS = CF + PI$

CF

$= \frac{\partial^2 y}{\partial x^2} + 4\partial y/\partial x + 5y = 0$

auxiliary equation becomes

$m^2 + 4m + 5 = 0$

using quadratic formula

$a = 1,\ b = 4,\ c = 4$

$$\frac{-4 \pm \sqrt{4^2 - (4 \times 1 \times 5}}{2 \times 1}$$

$$= \frac{-4 \pm \sqrt{16-20}}{2}$$

$\sqrt{-4} = \sqrt{4} \times \sqrt{-i}$

$= 2j$

$$\frac{-4 \pm 2i}{2}$$

$= -2 \pm j$

comparing with $m = \alpha + \beta j$

$\alpha = -2 \quad \beta = 1$

$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

$y = e^{-2x}(A\cos x + B\sin x)$

Particular Integral

Assumed PI

$y = Ce^{-2x}$

But since $Ce^{-2x}$ is same in CF & PI

multiply through by $x$

$y = xCe^{-2x}$

$y = Cxe^{-2x}$ - ①

$\frac{\partial y}{\partial x} = -2Cxe^{-2x}$ - ②

$\frac{\partial^2 y}{\partial x^2} = 4Cxe^{-2x}$ - ③

sub equ ①, ②, ③ into the general equation

$\frac{\partial^2 y}{\partial x^2} + 4\frac{\partial y}{\partial x} + 5y = 2e^{2x}$

$4Cxe^{-2x} + 4(-2Cxe^{-2x})$

$+ 5(Cxe^{-2x}) = 2Ce^{-2x}$

$4Cxe^{-2x} - 8Cxe^{-2x} + 5Cxe$

$= 2Ce^{-2x}$

$Cxe^{-2x}(1) = 2Ce^{2x}$

$Cx = 2$

$C = 2/x$ - ⑥

Sub equ ⑥ into ①

$y = Cxe^{-2x}$

$y = \frac{2}{x} xe^{-2x}$

The general solution becomes

$y = e^{-2x}(A\cos x + B\sin x)$

$+ 2e^{-2x}$

values for A & B

when $x = 0, y = 1$

Scanned by CamScanner

(5) $\frac{\partial^2 y}{\partial x^2} - 2\frac{\partial y}{\partial x} + y = 4\sin x$

CF: $\frac{\partial^2 y}{\partial x^2} - 2\frac{\partial y}{\partial x} + y = 0$

auxilary equation becomes

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$m = 1$ (twice)

$CF = e^x (A + Bx)$

Particular integral (PI)

Assumed PI

$y = C\cos x + D\sin x$ - (1)

$\frac{\partial y}{\partial x} = -C\sin x + D\cos x$ - (2)

$\frac{\partial^2 y}{\partial x^2} = -C\cos x - D\sin x$ - (3)

sub (1), (2), (3) into orignal equation

$\frac{\partial^2 y}{\partial x^2} - 2\frac{\partial y}{\partial x} + y = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$= 2C\sin x - 2D\cos x = 4\sin x$

comparing the coefficient

$2C = 4$

$-2D = 0$

$C = 4/2$ - (4)

$D = 0$ - (5)

$y = C\cos x + D\sin x$

$y = 2\cos x + C\sin x$

$y = 2\cos x$ - (6)

general solution:

$CF + PI$

$y = e^x (A + Bx) + 2\cos x$

$Ce^{-2x} = e^{-2x}$

$C = 1$

$\therefore$ ~~$y = Ce^{-2x}$~~ $y = Ce^{-2x}$

$y = e^{-2x}$

General Solution = CF + PI

$y = e^{-x}(A + Bx) + e^{-2x}$.

(4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

(1) (CF)

Auxilary equation becomes

$\frac{d^2y}{dx^2} + 25y = 0$

$m^2 + 25 = 0$

$m^2 = -25$

$m = \pm\sqrt{-25}$

$m = \pm\sqrt{25} \cdot \sqrt{-1}$

$m \pm 5$; Complex root)

$\therefore$ CF comparing with m

$= \alpha \pm \beta j$

$\alpha = 0$ $\beta = 5$

$C \cdot F = e^{0x}(A\cos 5x + B\sin 5x)$

Since $= e^0 = 1$

(2) PI

= Assumed PI

$y = Cx^2 + Dx + E$ ... (1)

$\frac{dy}{dx} = 2C + D$ ... (2)

$\frac{d^2y}{dx^2} = 2C$ ... (3)

Sub eqn (1), (2), (3) into

the general equation

$d^2y/dx^2 + 25y = 5x^2 + x$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25Cx^2 = 5x$

$25C = 5$

$C = \frac{5}{25}$

$C = 1/5$ — (4)

$25Dx = x$

$25D = 1$

$D = \frac{1}{25}$ — (5)

$2C + 25E = 0$

recall $C = 1/5$ (4)

$2(1/5) + 25E = 0$

$2/5 + 25E = 0$

$25E = 2/5$

$E = 2/5 \times 1/25$

$E = 2/125$ ... (6)

Sub equ (4), (5) & (6) into eqn (1)

$y = Cx^2 + Dx + E$

$y = \frac{1}{5}x^2 + 1/25x + 2/125$

general Solution =

CF + PI

$= y = A\cos 5x + B\sin 5x + (1/5x^2$

$1/25x + \frac{2}{125})$.

15|ENG04|043
Kafi Mobolaji
Mechanical Engineering

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

recall
For ~~hom~~ non-homogenous equation
General Solution = Complementry function (CF) + Particular integral (P.I)

: Complementary function (C.F)

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

the auxilary equation becomes

$$m^2 - m - 2 = 0$$

$$m^2 - 2m + m - 2 = 0$$

$$m(m-2) + 1(m-2) = 0$$

$$m+1 = 0 \ \& \ m-2 = 0$$

$$m = 1 \ \& \ m = 2 \rightarrow \text{real \& Different roots}$$

PI

$y = C$ - (1)

$\frac{dy}{dx} = 0$ (2)

$\frac{d^2y}{dx^2} = 0$ - (3)

Sub equ (1) - (2) - (3) into general equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$$

Scanned by CamScanner

$Ce^{-2x} = e^{-2x}$

$C = 1$

$\therefore \cancel{y = Ce^{-2x}}$ $y = Ce^{-2x}$

$y = e^{-2x}$

General Solution = CF + PI

$y = e^{-x}(A + Bx) + e^{-2x}$.

(4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

(1) (CF)

Auxilary equation becomes

$\frac{d^2y}{dx^2} + 25y = 0$

$m^2 + 25 = 0$

$m^2 = -25$

$m = \pm\sqrt{-25}$

$m = \pm\sqrt{25} \cdot \sqrt{-1}$

$m \pm 5$; (complex root)

$\therefore$ CF comparing with $m$

$= \alpha \pm \beta j$

$\alpha = 0$ $\beta = 5$

$C.F = e^{0x}(A\cos 5x + B\sin 5x)$

Since $= e^0 = 1$

(2) PI

= Assumed PI

$y = Cx^2 + Dx + E$ (1)

$\frac{dy}{dx} = 2C + D$ (2)

$\frac{d^2y}{dx^2} = 2C$ (3)

Sub eqn (1), (2), (3) into the general equation

$d^2y/dx^2 + 25y = 5x^2 + x$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25Cx^2 = 5x$

$25C = 5$

$C = \frac{5}{25}$

$C = 1/5$ — (4)

$25Dx = x$

$25D = 1$

$D = \frac{1}{25}$ — (5)

$2C + 25E = 0$

recall $C = 1/5$ (4)

$2(1/5) + 25E = 0$

$2/5 + 25E = 0$

$25E = 2/5$

$E = 2/5 \times 1/25$

$E = 2/125$ — (6)

Sub equ (4), (5) & (6) into eqn (1)

$y = Cx^2 + Dx + E$

$y = \frac{1}{5}x^2 + 1/25x + 2/125$

general Solution =

CF + PI

$= y = A\cos 5x + B\sin 5x + (1/5x^2 + 1/25x + \frac{2}{125})$.