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Matric : 16/ENG04/053

1) $$\lim_{x \to \frac{\pi}{2}} \left[\left(x^2 - \frac{\pi}{4}\right) \sin(\cos x) \right]$$

$$= \left[\frac{\left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4}\right) \sin(\cos \frac{\pi}{2})}{\pi/2 - \pi/2} \right]$$

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Using L'Hopital's rule $U \frac{dy}{dx} + v \frac{dy}{dx}$

$\frac{dy}{dx}$ = let $U = x^2 - \pi/4$ and $v = \sin(\cos x)$

$dy/dx = \sin(\cos x)$ = Let $\cos x = w$

$U = \sin x$

$dy/dy = \cos o$, $\frac{dw}{dx} = -\sin x$

$\frac{dy}{dx} = \frac{dy}{dw} \times \frac{dw}{dx} = -\sin x \cos(\cos x)$

$$= \left(x^2 - \frac{\pi}{4}\right) x - \sin x \cos(\cos x) + \sin(\cos x)(2x)$$

$\left(\frac{\pi}{2}\right)^2 - \pi/4 \, x - \sin 90 \cos(\cos 90) + \sin \cos 90 \times 2(\pi/2)$

$$= \left(\frac{\pi^2}{4} - \frac{\pi}{4}\right) x - 1 + 0 \times \pi$$

2) Determine whether each of the following series is convergent

a) $\frac{2}{2\times 3} + \frac{2}{3\times 4} + \frac{2}{4\times 5} + \frac{2}{5\times 6} + \ldots$

$u_n = \frac{2}{(n+1)(n+2)}$ $\qquad u_{n+1} = \frac{2}{(n+2)(n+3)}$

$\frac{u_{n+1}}{u_n} = \frac{2}{(n+2)(n+3)} \times \frac{(n+1)(n+2)}{2}$

$= \frac{n+1}{n+3}$

$\lim \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{n+1}{n+3}$

$= \frac{\frac{n}{n} + \frac{1}{n}}{\frac{n}{n} + \frac{3}{n}} = \frac{1+\frac{1}{n}}{1+\frac{3}{n}} = \frac{1+0}{1+0} = \frac{1}{1} = 1$

Since $\lim_{n\to\infty} \frac{u_{n+1}}{u_{n+1}}$

The Series is inconclusive

2b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \ldots$

$u_n = \frac{2}{n^2}$ $\qquad u_{n+1} = \frac{2}{(n+1)^2}$

$\lim_{n\to\infty} = \frac{u_{n+1}}{u_n} = \frac{2}{(n+1)^2} \times \frac{n^2}{2}$

$= \frac{n^2}{(n+1)^2} = \frac{n^2}{n^2+2n+1}$

$\lim = \frac{\frac{n^2}{n^2}}{\frac{n^2}{n^2} + \frac{2n}{n^2} + \frac{1}{n^2}} = \frac{1}{1+\frac{2}{n}+\frac{1}{n^2}} = \frac{1}{1} = 1$

$$\frac{\left(8 + \frac{2}{n} + \frac{1}{n^2} + \frac{1}{n^3}\right)}{\left(8 + \frac{12}{n} + \frac{24}{n^2} + \frac{8}{n}\right)}$$

as $n \to \infty$, $\frac{1}{n} \to 0$

$$\frac{8x}{8} \geq x - 1 \quad x < 1$$

Evalute using L'Hopital's Rule

$$\lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3}\right)$$

$$\lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3}\right) = \lim_{x \to 0} \left(\frac{\cos x + \sin x}{3x^2}\right)$$

$$= \lim_{x \to 0} \left(\frac{-\sin x + \cos x}{6x}\right)$$

$$= \lim \left(\frac{-\cos x - \sin x}{6}\right)$$

$$= \frac{-\cos 0 - \sin 0}{6}$$

$$= \frac{-1 - 0}{6}$$

$$= -\frac{1}{6}$$