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15/ENG04/027
MECHATRONICS

1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

The above equation can also be written as

$y'' - y' - 2y = 8$

let $y = e^{kx}$; $y' = ke^{kx}$; $y'' = k^2e^{kx}$

$k^2e^{kx} - ke^{kx} - 2e^{kx} = 8$

$(k^2 - k - 2)(e^{kx}) = 8$

solve as homogeneous

$k^2 - k - 2 = 0$

~~$(k^2 - 2k) + (k - 2) = 0$~~ $\quad (k^2 + k) - (2k - 2) = 0$

~~$k(k-2)\ 2$~~ $\quad k(k+1) - 2(k+1) = 0$

$(k-2)(k+1) = 0$

$k = 2$ or -1

$\therefore k_1 = 2 \quad k_2 = -1$

~~$y_1 = C_1$~~

$y_h = C_1y_1 + C_2y_2 \quad y_1 = e^{k_1x} = e^{2x}$

$y_h = C_1e^{2x} + C_2e^{-x} \quad y_2 = e^{k_2x} = e^{-x}$

Let $y = c$

then, $y' = 0$ and $y'' = 0$

substitute into original equation

$0 - 0 - 2y = 8$

$-2y = 8$

$y_p = -4$

$y = y_h + y_p$

$\therefore y = C_1e^{2x} + C_2e^{-x} + (-4)$

2. $\frac{d^2y}{dx^2} - \cancel{\frac{dy}{dx}} - 4y = 10e^{3x}$

$y'' - 4y = 0$ (solving for y_h)

$y'' = k^2e^{kx} \quad y' = ke^{kx} \quad ; y = e^{kx}$

$k^2e^{kx} - 4e^{kx} = 0$

$(k^2-4)e^{kx}=0$

$k^2-4=0$

$k=\pm\sqrt{4}$

$k=\pm 2$ i.e. $k_1=+2$; $k_2=-2$

$y_h = C_1\cosh 2x + C_2\sinh 2x$

y_p For y_p

$y=Ae^{+3x}$; $y'=+3Ae^{+3x}$; $y''=+9Ae^{+3x}$

Substitute into original equation

$9Ae^{+3x}-4(Ae^{+3x})=10e^{3x}$

$9Ae^{3x}-4Ae^{3x}=10e^{3x}$

$5Ae^{3x}=10e^{3x}$

$5A=10$

$A=2$

$y=y_h+y_p = C_1\cosh 2x + C_2\sinh 2x + 2e^{3x}$

3. $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=e^{-2x}$

For y_h

$(k^2+2k+k)e^{kx}=e^{-2x}$

$k^2+2k+k=0$

$(k^2+k)+(k+k)=0$ multiply through by 2

$k(k+1)+k($ $2k^2+4k+2k=0$

$\frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-2\pm\sqrt{2^2-(4\times1\times1)}}{2\times1} = -2\pm\sqrt{\ }$

3. $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=e^{-2x}$

solving for y_h

$(k^2+2k+1)(e^{kx})=0$

$k^2+2k+1=0$

$(k^2+k)+(k+1)=0$

$k(k+1)+1(k+1)=0$

$(k+1)(k+1)=0$

$k=-1$ twice

$y_h = C_1 e^{k_1 x} + C_2 x e^{k_2 x}$

$y_h = C_1 e^{-x} + C_2 x e^{-x}$

For y_p

$y = Ae^{-2x}$; $y' = -2Ae^{-2x}$; $y'' = 4Ae^{-2x}$

substitute in original equation

$4Ae^{-2x} + 2(-2Ae^{-2x}) + Ae^{-2x} = e^{-2x}$

$4Ae^{-2x} - 4Ae^{-2x} + Ae^{-2x} = e^{-2x}$

$Ae^{-2x} = e^{-2x}$

$A = 1$

$y = y_h + y_p = C_1 e^{-x} + C_2 x e^{-x} + e^{-2x}$

A. $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$y'' + 25y = 5x^2 + x$

solving for y_h

$y'' + 25y = 0$

$k^2 e^{kx} + 25e^{kx} = 0$

$k^2 + 25 = 0$

$k = \pm\sqrt{-25}$

$k = \pm 5i$ i.e. $k_1 = 5i$; $k_2 = -5i$

$y_h = C_1 \cos 5x + C_2 \sin 5x$

Solving for y_p

$y_p = Ax^2 + Bx + C$

$y'_p = 2Ax + B$

$y''_p = 2A$

substitute into original equation

$2A + 25(Ax^2 + Bx + C) = 5x^2 + x$

$2A + 25Ax^2 + 25Bx + 25C = 5x^2 + x + 0$

x^2 : $25A = 5$; x : $25B = 1$; $2A + 25C = 0$

$A = \frac{1}{5}$; $B = \frac{1}{25}$; $2(\frac{1}{5}) + 25C = 0$

$25C = -\frac{2}{5}$

$C = -\frac{2}{125}$

$$y_p = \frac{x^2}{5} + \frac{x}{25} - \frac{2}{125}$$

$$y = y_h + y_p = C_1 \cos 5x + C_2 \sin 5x + \frac{x^2}{5} + \frac{x}{25} - \frac{2}{125}$$

5. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$y'' - 2y' + y = 4\sin x$

solving for y_h

$k^2 e^{kx} - 2k e^{kx} + e^{kx} = 0$

$(k^2 - 2k + 1)(e^{kx}) = 0$

$k^2 - 2k + 1 = 0$

$(k^2 - k)(k + 1) = 0$

$k(k-1) - 1(k-1) = 0$

$(k-1)(k-1) = 0$

$k = 1$ twice

$y_h = C_1 e^x + C_2 x e^x$

solving for y_p

$y_p = A\sin x + B\cos x$

$y'_p = A\cos x - B\sin x$

$y''_p = -A\sin x - B\cos x$

substitute into original equation

$(-A\sin x - B\cos x) - 2(A\cos x - B\sin x) + (A\sin x + B\cos x) = 4\sin x + 0\cos x$

$-A\sin x - B\cos x - 2A\cos x - 2B\sin x + A\sin x + B\cos x = 4\sin x + 0\cos x$

$-2A\cos x - 2B\sin x = 4\sin x + 0\cos x$

$\sin x : -2B = 4$; $\cos x : -2A = 0$

$B = -2$; $A = 0$

$y_p = 2\sin x \cancel{-2\cos x}$

$y = y_h + y_p = C_1 e^x + C_2 x e^x \cancel{-2\sin x} - 2\cos x + 2\sin x$

6. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$ $y(0) = 1$ and $\frac{dy}{dx}(0) = -1$

$y'' + 4y' + 5y = 2e^{-2x}$

solving for y_h

$k^2e^{kx} + 4ke^{kx} + 5e^{kx} = 0$

$(k^2 + 4k + 5)(e^{kx}) = 0$

$k^2 + 4k + 5 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - (4 \times 1 \times 5)}}{2 \times 1} = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2i}{2}$

$k = -2 \pm i$ i.e. $k_1 = -2 + i$; $k_2 = -2 - i$

$y_h = C_1 e^{(-2+i)x} + C_2 e^{(-2-i)x}$

$y_h = C_1 e^{-2x} e^{ix} + C_2 e^{-2x} e^{-ix}$

$y_h = e^{-2x} [C_1 e^{ix} + C_2 e^{-ix}]$

For y_p

$y = Ae^{-2x}$; $y' = -2Ae^{-2x}$; $y'' = 4Ae^{-2x}$

substitute to original equation

$4Ae^{-2x} + 4(-2Ae^{-2x}) + 5Ae^{-2x} = 4e^{-2x}$

$4Ae^{-2x} - 8Ae^{-2x} + 5Ae^{-2x} = 4e^{-2x}$

$Ae^{-2x} = 4e^{-2x}$

$A = 4$

$y_p = 4e^{-2x}$ $\quad y = y_h + y_p = e^{-2x}[C_1 e^{ix} + C_2 e^{-ix}] + 4e^{-2x}$

$y_h = C_1 e^{-2x} e^{ix} + C_2 e^{-2x} e^{-ix}$ —(i)

$y' = C_1 \cdot -2e^{-2x} \cdot ie^{ix} + C_2 \cdot -2e^{-2x} \cdot -ie^{-ix}$ —(ii)

at $x = 0$, $y = 1$, $y' = -2$

from eqn (i)

$1 = C_1 e^0 e^0 + C_2 e^0 e^0$

$1 = C_1 + C_2 \Rightarrow C_1 = 1 - C_2$ —(iii)

from eqn (ii)

$-2 = C_1 \cdot -2 \cdot i + C_2 \cdot -2 \cdot -i$

$-2 = -2C_1 i + 2C_2 i$

$-1 = C_1 - C_2$ —(iv)

Substitute value of C_1 into (iv)

$-1 = -(1 - C_2) - C_2$

$-1 = -1 + C_2 - C_2$

$y = C_1 e^{-2x} e^{ix} + C_2 e^{-2x} e^{-ix} + 4e^{-2x}$ —(i)

$y' = C_1 [illegible] e^{-2x} e^{ix} + C_2 [illegible] e^{-2x} e^{-ix} - 8e^{-2x}$ —(ii)

at $x = 0$, [illegible]

$1 = C_1 + C_2 + 4 \Rightarrow C_1 = -3 - C_2$ (iii)

from (ii)

$-2 = [illegible]$ —(iv)

substitute for C_1 in (iv)

$-1 = [illegible]$

$C_2 = [illegible]$

substitute for C_2 in (iii)

$C_1 = [illegible]$

$\therefore y = e^{-2x}[e^{ix} + [illegible] e^{-ix}] + 4e^{-2x}$

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7. $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3y'' - 2y' - y = 2x - 3$

For y_h

$3k^2e^{kx} - 2ke^{kx} - e^{kx} = 0$

$(3k^2 - 2k - 1)e^{kx} = 0$

$3k^2 - 2k - 1 = 0$

$(3k^2 - 3k) + (k - 1) = 0$

$3k(k-1) + 1(k-1) = 0$

$(3k+1)(k-1) = 0$

$k = -\frac{1}{3}$ or 1

$k_1 = -\frac{1}{3}$ or $k_2 = 1$

$y_h = C_1e^{-\frac{1}{3}x} + C_2e^{x}$

For y_p

$y_p = Ax + B$

$y'_p = A$

$y''_p = 0$

substitute into original equation

$0 - 2A - Ax - 3 = 2x - 3$

$-2A - Ax - B = 2x - 3$

$x: -A = 2$; $-2A - B = -3$

$A = -2$; $-2(-2) - B = -3$

$B = 4 - 3 = 7$

$y_p = -2x + 7$

$y = y_h + y_p = C_1e^{-\frac{1}{3}x} + C_2e^{x} + (-2x + 7)$

8. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$y'' - 6y' + 8y = 8e^{4x}$

solving for y_h

$k^2e^{kx} - 6ke^{kx} + 8e^{kx} = 0$

$k^2 - 6k + 8 = 0$

$(k^2 - 4k) - (2k + 8) = 0$

$k(k-4) - 2(k-4) = 0$

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$(k-2)(k-4)=0$

$k = 2$ or 4 i.e. $k_1 = 2$; $k_2 = 4$

$y_h = C_1 e^{2x} + C_2 e^{4x}$

For y_p

$y_p = Ae^{4x}$

$y'_p = 4Ae^{4x}$

$y''_p = 16Ae^{4x}$

Substitute into original equation

$16Ae^{4x} - 6(4Ae^{4x}) + 8Ae^{4x} = 8e^{4x}$

$16Ae^{4x} - 24Ae^{4x} + 8Ae^{4x} = 8e^{4x}$

$22Ae^{4x} = 8e^{4x}$

$22A = 8$

$A = 4/11$

$y_p = \frac{4}{11} e^{4x}$

$y = y_h + y_p = C_1 e^{2x} + C_2 e^{4x} + \frac{4}{11} e^{4x}$