HABIB AHMAD ALKALI

15/ENG04/027

MECHATRONICS

2.(i) $y = x^3 e^{4x}$

solution;

Let $\omega = x^3 e^{4x}$

$v = x^3$; $v' = 3x^2$; $v'' = 6x$, $v''' = 6$, $v^{iv} = 0$

$u = e^{4x}$; $u' = 4e^{4x}$; $u'' = 16e^{4x}$; $u''' = 64e^{4x}$; $u^n = 4^n e^{4x}$

$$\therefore \omega^n = u^n v + \frac{n u^{n-1} v'}{1} + \frac{n(n-1) u^{n-2} v''}{1\times 2} + \frac{n(n-1)(n-2) u^{n-3} v'''}{1\times 2\times 3} +$$

$$\omega^n = 4^n e^{4x} x^3 + \frac{n 4^{n-1} e^{4x} 3x^2}{1} + \frac{n(n-1) 4^{n-2} e^{4x} 6x}{1\times 2} + \frac{n(n-1)(n-2) 4^{n-3} e^{4x} 6}{1\times 2\times 3}$$

$$\omega^n = 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + n(n-1) 4^{n-2} e^{4x} 3x + n(n-1)(n-2) 4^{n-3} e^{4x}$$

$$y^5 = \omega^5 = (4^5 e^{4x} x^3) + (5\times 4^4 \times e^{4x} 3x^2) + (5(5-1) 4^3 e^{4x} 3x) + (5(4)(3) 4^2 e^{4x})$$

$$= 1024 e^{4x} x^3 + 1280 e^{4x} 3x^2 + 1280 e^{4x} 3x + 960 e^{4x}$$

(ii) $x^2 \, d^2y/dx^2 + x \, dy/dx + y = 0$

solution.

the equation above can be written as

$$x^2 y'' + x y' + y = 0$$

Let $\omega = x^2 y''$

$v = x^2$; $v' = 2x$, $v'' = 2$, $v''' = 0$

$u = y''$; $u' = y'''$; $u'' = y^{iv}$; $u^n = y^{n+2}$

$$\omega^n = y^{n+2} x^2 + n y^{n+1} 2x + \frac{n(n-1) y^n 2}{2}$$

$$\omega^n = y^{n+2} x^2 + n y^{n+1} 2x + n(n-1) y^n$$

Let $\omega = x y'$

$v = x$; $v' = 1$; $v'' = 0$

$u = y'$; $u' = y''$; $u^n = y^{n+1}$

$$\omega^n = y^{n+1} x + n y^n$$

Scanned by CamScanner

Re-writing the original equation, we have

$y^{n+2}x^2 + 2nx\,y^{n+1} + n(n-1)y^n + x\,y^{n+1} + ny^n + y^n = 0$

$\cancel{y^2 x^2 y^{n+2} + [2nx\,y^{n+1} +}$

$x^2 y^{n+2} + [2nx + x]y^{n+1} + [n(n-1) + n + 1]y^n = 0$

$x^2 y^{n+2} + [2nx + x]y^{n+1} + [n^2+1]y^n = 0$

$x^2 y^{n+2} + (2n+1)x\,y^{n+1} + (n^2+1)y^n = 0$

1. $y = e^{x^2+x}$ show that $y'' = y'(2x+1) + 2y$

solution:

$y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = [(2x+1) \times (2x+1)e^{x^2+x}] + [e^{x^2+x} \times 2]$ ← Product Rule

$= 4x^2 + 4x + 2\,e^{x^2+x} + 2e^{x^2+x}$

$= 4x^2 + 4x + 4\,e^{x^2+x}$

equating equations

$4x^2 + 4x + 4\,e^{x^2+x} = y'(2x+1) + 2y$

$= (2x+1)e^{x^2+x}(2x+1) + 2(e^{x^2+x})$

$= 4x^2 + 4x + 2\,e^{x^2+x} + 2e^{x^2+x}$

$= 4x^2 + 4x + 4\,e^{x^2+x}$

$\therefore y'' = y'(2x+1) + 2y$ — (1)

Hence, prove that, $y^{(n+2)} = (2x+1)y^{n+1} + 2(n+1)y^n$

Rewriting equation (1) we have,

$y'' - y'(2x+1) - 2y = 0$

Let $w = y''$

$v = 1$, $v' = 0$

$u = y''$, $u' = y'''$, $u^n = y^{n+2}$

$w^n = y^{n+2} + ny^{n+1}\,0$

$= y^{n+2}$

Let $w = (2x+1)y'$

$v = (2x+1)$, $v' = 2$, $v'' = 0$

$u = y'$, $u' = y''$, $u^n = y^{n+1}$

$w^n = y^{n+1}(2x+1) + n\,y^{n}\,2 = y^{n+1}(2x+1) + 2n\,y^{n}$

Substituting

$$y^{n+2} - \left[(2x+1)y^{n+1} + 2ny^n\right] - 2y^n = 0$$

$$y^{n+2} - (2x+1)y^{n+1} - 2ny^n - 2y^n = 0$$

$$y^{n+2} - (2x+1)y^{n+1} - \cancel{2(n+1)y^n} \quad 2(n+1)y^n$$

$$y^{n+2} = (2x+1)y^{n+1} + 2(n-1)y^n$$