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15/ENG04/027

MECHATRONICS

1. (i) $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

$y'' + 4y' + 5y = 6\sin\theta$

$k^2 + 4k + 5 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - (4\times1\times5)}}{2\times1} = \frac{-4 \pm \sqrt{-4}}{2}$

$= \frac{-4 \pm 2i}{2} = -2 \pm i$

$k_1 = -2 + i$, $k_2 = -2 - i$

$y = C_1 y_1 + C_2 y_2$

$y_1 = e^{k\theta} = e^{(2+i)\theta}$

$y_2 = e^{-k_2\theta} = e^{-2\theta(2-i)\theta}$

$y = C_1 e^{(-2+i)\theta} + C_2 e^{(-2-i)\theta}$

$y = C_1 e^{-2\theta} e^{i\theta} + C_2 e^{-2\theta} e^{-i\theta}$

$y_h = e^{-2\theta}\left[C_1 e^{i\theta} + C_2 e^{-i\theta}\right]$

~~$y_h = C_1\cos 2\theta + C_2\sin 2\theta$~~

$y_h = e^{-2\theta}\left[A\cos\theta + B\sin\theta\right]$

(ii) $y_p = A\cos x + B\sin x$

$y_p' = -A\sin x + B\cos x$

$y_p'' = -A\cos x - B\sin x$

$(-A\cos x - B\sin x) + 4(-A\sin x + B\cos x) + 5(A\cos x + B\sin x) = 6\sin x + 0\cos x$

$-A\cos x - B\sin x - 4A\sin x + 4B\cos x + 5A\cos x + 5B\sin x = 6\sin x + 0\cos x$

$(-A + 4B + 5A)\cos x + (-B - 4A + 5B)\sin x = 6\sin x + 0\cos x$

$\cos x$: $-A + 4B + 5A = 0$

$4B + 4A = 0$

$A = -B$

$\sin x$: $-B - 4A + 5B = 6$

$4B - 4A = 6$

$4B - 4(-B) = 6$

$4B + 4B = 6$

$8B = 6$

$B = \frac{6}{8} = \frac{3}{4}$

$A = -B$

$A = -\frac{3}{4}$

$y_p = -\frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

~~$y = y_n + y_p = e^{-2\theta}[C_1 e^{i\theta} + C_2 e^{-i\theta}] + [-\frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta]$~~

$y = y_n + y_p = e^{-2\theta}[A\cos\theta + B\sin\theta] + [-\frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta]$

(iii) At steady state

$\theta = \infty$ $\quad\quad dy/d\theta = 0$

$$\frac{dy}{d\theta} = \frac{d(y)}{d\theta}$$

$u = e^{-2\theta}$ $\quad\quad V = A\cos\theta + B\sin\theta$

$du = -2e^{-2\theta}$ $\quad\quad dv = -A\sin\theta + B\cos\theta$

$e^{-2\theta}[-A\sin\theta + B\cos\theta] + [A\cos\theta + B\sin\theta] - 2e^{-2\theta}$ $\frac{d(y)}{d\theta}$

$dy/d\theta = e^{-2\theta}[-A\sin\theta + B\cos\theta] + [A\cos\theta + B\sin\theta] - 2e^{-2\theta} + \frac{3}{4}\sin\theta$
$+ \frac{3}{4}\cos\theta$

at steady state,

$\theta = \infty$ $\quad\quad dy/d\theta = 0$

~~$\theta = \frac{3}{4}\cos$~~

$0 = \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$

Divide through by $\cos\theta$

$0 = \frac{3}{4}\tan\theta + \frac{3}{4}$

$\frac{3}{4}\tan\theta = -\frac{3}{4}$

$\tan\theta = -1$

$\theta = \tan^{-1}(-1)$

$\theta = -45°$

2. $EI\, d^2y/dx^2 = \frac{w}{2}(L-x)^2$

$$EI\frac{dy}{dx^2} = 0$$

Let $\frac{d^2y}{dx^2} = K^2$

$EI(K^2) = 0$

$K^2 = 0$ $\quad\quad K = 0$

$\cancel{y_h = A\cos\theta + B\sin\theta}$

$\cancel{y_h = A}$

$y_h = e^{0}(A + Bx)$

$y_h = A + Bx$

Particular function

$$y = \frac{w}{2}(L-x)^2$$

$$y = Cx^2 + Dx + E$$

$$\frac{d^2y}{dx^2} = 2C$$

$$EI(2C) = \frac{w}{2}(L-x)^2$$

$$EI(2C) = \frac{wL^2}{2} - \frac{2xLw}{2} + \frac{wx^2}{2}$$

Particular function

$$y = \frac{w}{2}(L-x)^2$$

$$y = Cx^2 + Dx^3 + Ex^4$$

$$\frac{dy}{dx} = 2Cx + 3Dx^2 + 4Ex^3$$

$$\frac{d^2y}{dx^2} = 2C + 6Dx + 12Ex^2$$

substitute into original equation

$$EI(2C + 6Dx + 12Ex^2) = \frac{w}{2}(L^2 - 2Lx + x^2)$$

$$2(2CEI + 6DEIx + 12EEIx^2) = w(L^2 - 2Lx + x^2)$$

$$4CEI + 12DEIx + 24EEIx^2 = w(L^2 - 2Lx + x^2)$$

Equating Coefficient

$$\Rightarrow w = 24E\,EI$$

$$E = \frac{w}{24EI} \quad \text{—— (1)}$$

$$\Rightarrow 12DEI = -2wL$$

$$D = \frac{-2wL}{12EI} = \frac{-wL}{6EI} \quad \text{—— (2)}$$

$\Rightarrow 4CEI = wL^3$

$$C = \frac{wL^3}{4EI} \quad \text{---} (2)$$

$$y = \left[\frac{wL^2}{4EI}\right]x^2 - \left[\frac{wL}{6EI}\right]x^3 + \left[\frac{w}{24EI}\right]x^4$$

$$y = \frac{wL^2x^2}{4EI} - \frac{wLx^3}{6EI} + \frac{wx^4}{24EI}$$

$$y = \frac{6wL^2x^2 - 4wLx^3 + wx^4}{24EI}$$

$$y_p = \frac{w}{24EI}\left(6L^2x^2 - 4Lx^3 + x^4\right)$$

$$y = y_c + y_p$$

$$y = A + Bx + \frac{w}{24EI}\left(6L^2x^2 - 4Lx^3 + x^4\right)$$

when $y = 0,\ x = 0,\ \frac{dy}{dx} = 0$

$$0 = A + B(0) + \frac{w}{24EI}\left(6L^2(0) - 4L(0) + 0\right)$$

$$0 = A + \quad \Rightarrow A = 0$$

for B

$$\frac{dy}{dx} = B + \frac{w}{24EI}\left(6L^2x^2 - 12Lx^2 + 4x^3\right)$$

$$0 = B + \frac{w}{24EI}\left(6L^2(0) - 12L(0) + 4(0)\right)$$

$$B = 0$$

$$\therefore y = A + Bx + \frac{w}{24EI}\left(6L^2x^2 - 4Lx^3 + x^4\right)$$

$$y = 0 + (0)x + \frac{w}{24EI}\left(6L^2L^2 - 4LL^3 + L^4\right)$$

$$y = \frac{w}{24EI}\left(6L^4 - 4L^4 + L^4\right)$$

$$y = \frac{w}{24EI}\left(3L^4\right)$$

$$y = \frac{wL^4}{8EI}$$

θ	y
0	-0.75
30	-0.27452
60	0.274519
90	0.75
120	1.024519
150	1.024519
180	0.75
210	0.274519
240	-0.27452
270	-0.75