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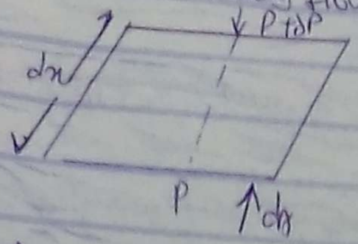
15/ENG01/012
CHEMICAL ENGINEERING.
CHE 311 ASSIGNMENT.

Using newton's second law, derive the bernoulli's equation

Consider a steady flow of an ideal fluid along a stream tube



Force on a fluid element

1- The pressure force in the direction of flow is given as

$F = PdA - (P + dP)dA$

$F = PdA - PdA - dPdA$

$F = -dPdA \quad \cdot (1)$

2- Compononent of the weight fluid vector is given as

$W = -\rho g\, dA\, ds \cos\theta \quad \cdot\cdot (2)$

$W = -\rho g\, dA \cdot \frac{dz}{ds}$

$W = \rho g\, dA \cdot dz \quad \cdot\cdot (3)$

3- mass of fluid element

$m = \rho dA \cdot ds \quad \cdot\cdot (4)$

4- Acceleration of fluid element

$a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt}$

$a = \frac{v \cdot dv}{ds} \quad \cdot\cdot (5)$

* since $F = ma$ $\cdot\cdot (6)$, according to the newtons second law

substitute equation 4 and 5 into 6

$F = \rho dA \cdot ds \times \frac{Vdv}{ds}$

$F = \rho dA \cdot Vdv \quad \cdot\cdot (7)$

Substitute equation 1 and 3 into 7

$-dpdA - \rho g dA dz = \rho dA \cdot v dv$

$(-dp - \rho g dz) = dA(\rho v dv)$

$-dp - \rho g dz = \rho v dv$

dividing through by ρ

$$\frac{-dp}{\rho} - g dz = v dv$$

$$g dz + v dv + \frac{dp}{\rho} = 0 \; \ldots \; [\text{at a constant density}]$$

~~equation 9 is the req~~

integrating eqn (9)

$$\frac{1}{\rho}\int dP + \int v dv + g\int dz = 0 \; \ldots (10)$$

$$P/\rho + V^2/2 + gz = 0 \; \ldots (11)$$

dividing through by g

$$P/\rho g + V^2/2g + z = 0 \; \ldots (12)$$

Recall; $w = \rho g$

$$P/w + \frac{v^2}{2g} + z = 0$$

Inlet $= \frac{P_1}{w} + \frac{V_1^2}{2g} + z_1$

outlet $= \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2$

hence Bernoullis equation

Sum of energy flowing in = sum of energy flowing out

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2$$