MGBOJIKWE CHUKWUEBUKA UGOCHUKWU
13/ENG01/007
Chemical Engineering
Process Dynamics Assignment

Problem Statement

Given that

$y(0) = 5$ and $\dot{y}(0) = 7$

Solve

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 2e^{3t}$$

Solution

With the knowledge that

$$\dot{y} = \frac{dy}{dt} \quad \text{and} \quad \ddot{y} = \frac{d^2y}{dt^2}$$

Hence,

$$\frac{d^2y}{dt^2} - \frac{3dy}{dt} + 2y = 2e^{3t}$$

$$\ddot{y} - 3\dot{y} + 2y = 2e^{3t}$$

Finding the Laplace of

i) $\frac{d^2y}{dt^2} \rightarrow \mathcal{L}\left[\frac{d^2y}{dt^2}\right] = s^2y(s) - sy(0) - \dot{y}(0)$

ii) $\frac{dy}{dt} \rightarrow \mathcal{L}\left[\frac{dy}{dt}\right] = sy(s) - y(0) \rightarrow 3[sy(s) - y(0)]$

iii) $y \rightarrow y(s)$

iv) $2e^{3t} \rightarrow \mathcal{L}[2e^{3t}] = 2\mathcal{L}[e^{3t}] = 2 \cdot \frac{1}{s-3}$

$$\rightarrow s^2y(s) - sy(0) - \dot{y}(0) - 3[sy(s) - y(0)] + 2y(s) = 2 \cdot \frac{1}{s-3}$$

Given $y(0) = 5$ and $\dot{y}(0) = 7$

$$\rightarrow s^2y(s) - 5s - 7 - 3sy(s) + 15 + 2y(s) = \frac{2}{s-3}$$

Collecting Like Terms

$$\rightarrow s^2y(s) - 3sy(s) + 2y(s) - 5s + 8 = \frac{2}{s-3}$$

$$y(s)[s^2 - 3s + 2] - 5s + 8 = \frac{2}{s-3}$$

Finding the LCM

$$y(s) = \frac{\frac{2}{s-3} + 5s + 8}{s^2 - 3s + 2} \rightarrow \frac{2}{s-3} + \frac{5s}{1} - \frac{8}{1} \cdot \frac{1}{s^2 - 3s + 2}$$

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$$y(s) = \frac{2 + 5s(s-3) - 8(s-3)}{s-3} \cdot \frac{1}{s^2-3s+2}$$

$$y(s) = \frac{2 + 5s^2 - 15s - 8s + 24}{(s-3)(s^2-3s+2)}$$

$$y(s) = \frac{5s^2 - 23s + 26}{(s-3)(s^2-3s+2)}$$

Using Calculator to Find Roots Of The Quadratic Equation In the Numerator and Denominator

$(s-2)$ $\quad s = \frac{13}{5} \rightarrow 5s = 13 \quad (5s-13)$

$(s-3) \quad (s-2) \quad (s-1)$

$x_2 = 2 \qquad x_1 = \frac{13}{5}$

$x_1 = 3 \qquad x_2 = 1 \qquad x_3 = 2$

$$y(s) = \frac{\cancel{(s-2)}(5s-13)}{(s-3)\cancel{(s-2)}(s-1)}$$

$$y(s) = \frac{5s-13}{(s-3)(s-1)}$$

Using PARTIAL FRACTIONS

$$\frac{5s-13}{(s-3)(s-1)} = \frac{A}{s-3} + \frac{B}{s-1}$$

$$\frac{5s-13}{(s-3)(s-1)} = \frac{A(s-1)+B(s-3)}{(s-3)(s-1)}$$

$\rightarrow \quad 5s - 13 = As - A + Bs - 3B$

$5s - 13 = (A+B)s - A - 3B$

Comparing Coefficients, $A+B = 5 \quad --- (1)$

$-A - 3B = -13$

$A + 3B = 13 \quad --- (2)$

Using Elimination Method

$A+B = 5$

$\underline{A+3B = 13}$

$-2B = -8$

$B = \underline{4}$

Substitute for B in equation (1)

$A + B = 5$

$A + 4 = 5$

$A = 1$

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Hence,

$$\frac{5s-13}{(s-3)(s-1)} = \frac{1}{s-3} + \frac{4}{s-1}$$

$y(t) =$

The Inverse Laplace Of The Above gives

$$y(t) = \mathcal{L}^{-1}\left[\frac{1}{s-3}\right] + \mathcal{L}^{-1}\left[\frac{4}{s-1}\right]$$

$$y(t) = \underline{\underline{e^{3t} + 4e^{t}}}$$

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