

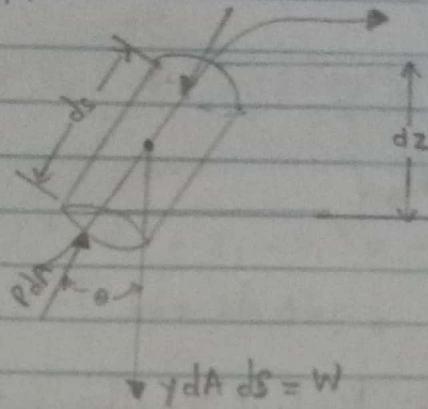
CHE 311

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MATRIC NO: 15/ENGO1/004

ASSIGNMENT 2

⇒ Derive the Bernoulli's Equation from the Newton second Law  
Let us consider a frictionless steady flow along a streamline.  
A cylindrical differential element as shown in the figure below



of length  $ds$  and cross sectional area  $dA$  is assumed along streamline.  
Intensity of pressures on the faces of the element are  $p$  and  $(p + dp)$ . So the pressure force acting on the element tending to acceleration fluid mass will be

$$p \cdot dA - (p + dp) \cdot dA = -dp \cdot dA \quad \text{--- (1)}$$

If  $\gamma$  be the unit weight of fluid mass, gravity force acting vertically downward will be  $(\gamma \cdot dA \cdot ds)$ . Component of gravity force along direction of motion will be

$$= -(\gamma \cdot dA \cdot ds) \cos \theta = -(\gamma \cdot dA \cdot ds) \frac{dz}{ds} = (\gamma \cdot dA \cdot dz) \quad \text{--- (2)}$$

The velocity being  $V$  along the direction of motion, acceleration for the steady flow will be  $\left( V \cdot \frac{dV}{ds} \right)$ . Applying Newton's Law of motion,

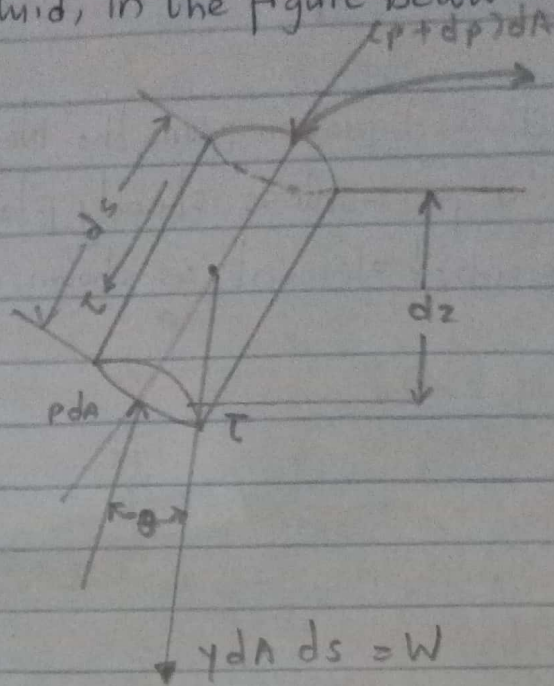
we have,

$$-dp \cdot dA - \gamma \cdot dA \cdot dz = \frac{\gamma}{g} \cdot dA \cdot ds \cdot V \frac{dV}{ds}$$

$$\Rightarrow \frac{dp}{\rho} + V dV + g dz = 0 \quad \text{--- (3)}$$

This equation is known as one-dimensional Euler's equation for an ideal fluid. It is applicable to both compressible and incompressible fluid flow.

For a real fluid, in the figure below



an additional term, force due to fluid friction will be effective. If  $\tau$  be the shear stress and  $r$  be the radius of the fluid element, the frictional force along the direction of motion will be  $(-2 \cdot \tau \cdot r \cdot ds)$

Modifying equation (3) we arrive at

$$-dp \cdot dA - \gamma \cdot dA \cdot dz - 2\tau r \cdot ds = \frac{\gamma}{g} dA \cdot ds \cdot v \frac{dv}{ds}$$

$$\Rightarrow \frac{dp}{\rho} + v dv + g dz = -\frac{2\tau \cdot ds}{r} \quad \text{--- 4}$$

This is Euler's equation for one dimensional real fluid flow.

To derive Bernoulli's equation from Euler's equation for the case of an incompressible ideal fluid where  $p = \text{constant}$ , equation (3) can be integrated as,

$$\int \frac{dp}{\rho} + \int v dv + \int g dz = \text{Constant}$$

$$\Rightarrow \frac{p}{\rho} + \frac{v^2}{2} + gZ = \text{Constant} \quad \text{--- (5)}$$

$$\Rightarrow \frac{p}{\rho} + \frac{v^2}{2g} + z = \text{Constant}$$

If the fluid becomes real and incompressible, integration over two section 1 and 2 of total fluid element length  $l$  yields

$$\int_1^2 \frac{dp}{\rho} + \int_1^2 V dv + \int_1^2 g dz = - \int_1^2 \frac{2\tau \cdot ds}{r}$$

$$\Rightarrow \frac{p_2 - p_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = - \frac{2\tau}{r} (l_2 - l_1)$$

$$\Rightarrow \frac{p_2 - p_1}{\gamma} + \frac{V_2^2 - V_1^2}{2g} + (z_2 - z_1) = - \frac{2\tau}{\gamma r} l \quad \text{--- (5)}$$

$$\Rightarrow \left( \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \right) - \left( \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \right) = \frac{2\tau l}{\gamma r}$$

applying equation (4.11) over two section 1 and 2, we have

$$\left( \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \right) - \left( \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \right) = 0$$

Hence right-hand side of equation (4.12) is designated as

$$h_f = \frac{2\tau \cdot L}{\gamma \cdot r} \quad \text{--- (6)}$$

This is the head-loss between two pre-selected sections in a conduit carrying real fluid. Equation (5) and (6) are popularly known as Bernoulli's equation for real and ideal fluid respectively.