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CHEMICAL ENGINEERING

CHE311

Define a parcel of fluid moving through a pipe with cross-sectional area *A*, the length of the parcel is d*x*, and the volume of the parcel *A* d*x*. If mass density is ρ, the mass of the parcel is density multiplied by its volume *m* = *ρA* d*x*. The change in pressure over distance d*x* is d*p* and [flow velocity](https://en.wikipedia.org/wiki/Flow_velocity) *v* = d*x*/d*t*.

Apply [Newton's second law of motion](https://en.wikipedia.org/wiki/Newton%27s_second_law_of_motion) (force = mass × acceleration) and recognizing that the effective force on the [parcel of fluid](https://en.wikipedia.org/wiki/Fluid_parcel) is −*A* d*p*. If the pressure decreases along the length of the pipe, d*p* is negative but the force resulting in flow is positive along the *x* axis.

m{\displaystyle {\begin{aligned}m{\frac {\mathrm {d} v}{\mathrm {d} t}}&=F\\\rho A\mathrm {d} x{\frac {\mathrm {d} v}{\mathrm {d} t}}&=-A\mathrm {d} p\\\rho {\frac {\mathrm {d} v}{\mathrm {d} t}}&=-{\frac {\mathrm {d} p}{\mathrm {d} x}}\end{aligned}}}$\frac{dy}{dx}\frac{dv}{dt}= F$

$$ρAdx\frac{dv}{dt}=-Adp$$

$$ρ\frac{dv}{dt}=-\frac{dp}{dx}$$

In steady flow the velocity field is constant with respect to time, *v* = *v*(*x*) = *v*(*x*(*t*)), so *v* itself is not directly a function of time *t*. It is only when the parcel moves through *x* that the cross sectional area changes: *v* depends on *t* only through the cross-sectional position *x*(*t*).

$$\frac{dv}{dt}=\frac{dv}{dx}.\frac{dx}{dt}$$

$$=\frac{dv}{dx}v$$

$$=\frac{d}{dx}(\frac{v^{2}}{2})$$

With density *ρ* constant, the equation of motion can be written as

$\frac{d}{dx}\left(ρ\frac{v^{2}}{2}+p\right)=0${\displaystyle {\frac {\mathrm {d} }{\mathrm {d} x}}\left(\rho {\frac {v^{2}}{2}}+p\right)=0}

by integrating with respect to *x*

$\frac{v^{2}}{2}+\frac{p}{ρ}=C${\displaystyle {\frac {v^{2}}{2}}+{\frac {p}{\rho }}=C}

where *C* is a constant, sometimes referred to as the Bernoulli constant. It is not a [universal constant](https://en.wikipedia.org/wiki/Universal_constant), but rather a constant of a particular fluid system. The deduction is: where the speed is large, pressure is low and vice versa.

In the above derivation, no external work–energy principle is invoked. Rather, Bernoulli's principle was derived by a simple manipulation of Newton's second law.