Zakari Farida
18/Engr/021
Chemical Engineering

① If $y = e^{ax^2 + x}$

$u = ax^2 + x$

$dy/dx = 2x + 1$

$y = e^u$

$dy/du = e^u$

$dy/dx = dy/du \times du/dx$

$= e^u \times 2x + 1$

$2x + 1 e^u \quad u = x^2 + x$

$dy/dx = 2x + 1 e^{x^2 + x}$

$d^2y/dx^2 = 2ex^2 + x$

$+ (2x + 1)(2x + 1) ex^2 + x$

$= 2e^{x^2 + x} + 4x^2 + 4x$

$+ 1 ex^2 + x$

$d^2y/dx^2 = 2e^{x^2 + x} + 4x^2$

$+ 4x + 1 e^{x^2 + x}$

$y'' = d^2y/dx^2 \quad y' = dy/dx$

$y = e^{x^2 + x}$

$y'' = y'(2x + 1) + 2y$

$y'' = 2e^{x^2 + x} + 4x^2 + 4x + ex^2 + x$

$y'(2x + 1) = (2x + 1)(2x + 1) e^{x^2 + x}$

$= 4x + 4x + e^{x^2 + x}$

$2y = 2e^{x^2 + x}$

$y'(2x + 1) + 2y = 2e^{x^2 + x} + 4x^2$

$+ 4x + ex^2 + x$

$y'' = 2e^{x^2 + x} + 4x^2 + 4x + e^{x^2 + x}$

$y'' = y'(2x + 1) + 2y$

$w_1 \quad w_2 \quad w_3$

$u = y'' \quad v = 1$

$u^n = y^{n+2} \quad v > 0$

$= y^{n+2} \cdot 1 + 0$

w_2

$u = y' \quad v = 2x + 1$

$u' = y^{n+1} \quad v' = 2$

$u^{n-1} = y^n \quad v = 0$

$y^{n+1}(2x + 1) + n(y^n) \cdot 2$

$= y^{n+1}(2x + 1) + 2n(y^n)$

w_3

$u = 2y \quad v = 1$

$u^n = y^n \quad v' = 0$

$= 2[(y^n - 1) + 0]$

$= 2y^n$

$w_1 = w_2 + w_3$

$y^{n+2} = y^{n+1}(2x + 1) + 2n$

$= y^{n+1}(2x + 1) + 2(n + 1)y^n$

2) Using the leibuitz theorem
that $y = xe^{4x}$ determine y^5
solution

$u = e^{4x} \quad v = x^3$

$y^{(5)} = u^{(5)}v + 5u^4v' + 10u^3v^2 + 10$

$+ 5u'v^4 + 4v^3$

$= 4^5 e^{4x} x^3 + 5(4^4 e^{4x} 3x^2) + 10$

$e^{4x} 6x) + 5(4^2 e^{4x} 6) + 0$

$= 1024 e^{4x} x^3 + 1280 e^{4x} 3x$

$+ 640 e^{4x} \cdot 6x + 80 e^{4x} \cdot 6x$

$= 1024 e^{4x} x^3 + 3840 e^{4x} x^2$

$3840 e^{4x} x + 480 e^{4x}$

3) $x^2 dy^2/dx^2 + x dy/dx + y$

$x^2 y'' + xy' + y = 0$

$w_1 \quad w_2 \quad w_3$

$w_1 + w_2 + w_3 = 0$

for w_1

$u = y'' \quad v = x^2$

$u^n = y^{n+2} \quad v' = 2x$

$u^{n-1} = y^{n+1} \quad v'' = 2$

$u^{n-2} = y^n \quad v''' = 0$

$= y^{(n+2)}(x^2) + n(y^{n+1})2x + \frac{n(n-1)y^n x + 0}{2!}$

$= x^2 y^{(n+2)} + 2nx(y^{n+1}) + n(n-1)y^n$

for w_2

$u = y^1 \qquad u = x$

$u^n = y^{n+1} \qquad v = 1$

$u^{n-1} = y^n \qquad v'' = 0$

$= y^{n+1} \cdot x + ny^n + 0$

for w_3

$u = y \quad v = 1 \quad u^n = y^n \quad v = 0$

$= y^n \cdot 1$

$w_1 + w_2 + w_3 = 0$

$x^2 y^{n+2} + 2nxy^{n+1} + (n^2 - n)y^n + xy^{n+1} + ny^n + y^n$

$xy^{n+2} + 2nxy^{n+1} + xy^{n+1} + n^2 y^n - ny^n + ny^n + y^n$

$xy^{n+2} + 2n+1(xy^{n+1}) + (n^2 + 1)y^n$