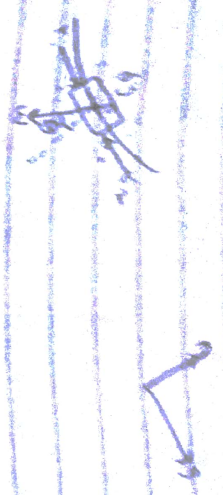


Assume flow along  
streamlines  
Laminar flow



$$\rho \dot{V} = \rho v A \Delta x$$

- Using the following assumptions
- 1. Inviscid fluid
- 2. Incompressible
- 3. Steady

$$\rho \dot{V} = \rho v A \Delta x \quad \text{I.e. constant } \rho$$

$$= \rho \dot{V} \Delta x \quad \text{I.e. constant } v$$

The component of weight force in direction of streamline

$$\Delta W_s = -\Delta W \sin \theta = -\rho v A \Delta x \sin \theta$$

$$= -\rho v \cdot v \Delta x$$

Let order Taylor series expansion for the pressure

$$\Delta p_s = \rho \frac{\partial p}{\partial x} \Delta x$$

net pressure force in streamline direction

$$\Delta F_p = (\rho \Delta p_s) \Delta x - (\rho \Delta p_s) \Delta x$$

$$= -\rho \frac{\partial p}{\partial x} \Delta x \Delta x$$

$$= -\rho \frac{\partial p}{\partial x} \Delta x$$

Hence, net force acting in streamline direction is

$$\sum dF_s = \partial W_s + \partial F_{ps}$$

$$= \left( -y \sin \theta - \frac{\partial p}{\partial s} \right) dv = -y \sin \theta - \frac{\partial p}{\partial s} \quad \dots (*)$$

$$= \rho v \frac{\partial v}{\partial s}$$

$$= \rho g_s$$

Since  $\sin \theta = \frac{dz}{ds}$  and  $v \frac{\partial v}{\partial s} = \frac{1}{2} \frac{\partial v^2}{\partial s}$

By rearranging and integrating (\*)

$$-y \frac{dz}{ds} - \frac{dp}{ds} = \frac{1}{2} \rho \frac{dv^2}{ds}$$

$$dp + \frac{1}{2} \rho d(v^2) + y dz = 0 \quad [\text{along a streamline}]$$

For constant acceleration of gravity:

$$\int \frac{dp}{\rho} + \frac{1}{2} v^2 + gz = C$$

For steady, inviscid and incompressible flow:

$$p + \frac{1}{2} \rho v^2 + \rho g z = C$$