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CHE 311
chemical Engineering.

Derive bernoulli's equation from Newton's Second law.

Although the bernoulli's equation is not concerned with the pressure gradient normal to the stream line, it does not mean that this pressure gradient is unimportant
Newton's second law states that the sum of the forces acting on a particle equals the particles mass time its acceleration.
Since we are tracking a particular fluid particle, Newton's Second law can be applied directly.

Mathematically:

$$\Sigma \vec{F} = m\vec{a}$$

Newton's Second law applied to the S-direction.

$$\Sigma F_s = ma_s$$

Step by Step derivation:

$$\Sigma F_s = m\frac{dv}{dt}$$

$$= \left[p - \frac{1}{2}\frac{dp}{ds}ds\right]dn\,dw - \left[p + \frac{1}{2}\frac{dp}{ds}\right]dn\,dw$$

$$- \rho g\,ds\,dn\,dw \sin\theta = -\left[\frac{dp}{ds} + \rho g \sin\theta\right] ds\,dn\,dw$$

$$m = \rho\,ds\,dn\,dw$$

$$\frac{dv}{dt} = \frac{dv(s,t)}{dt} = \frac{\partial v}{\partial t} \quad \frac{\partial v}{\partial s}\frac{\partial s}{\partial t} = v\frac{dv}{ds}$$

$$m\frac{dv}{dt} \quad \rho\,ds\,dn\,dw\left(v\frac{dv}{ds}\right)$$

$$\sin\theta = \frac{dz}{ds}$$

$$\rho v\frac{dv}{ds} = -\frac{dp}{ds} - \rho g\frac{dv}{ds}$$

$$\rho v\frac{dv}{ds} + \frac{dp}{ds} + \rho g\frac{dz}{ds} = 0$$

$$v\frac{dv}{ds} + \frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} = 0$$

$$v\,dv + \frac{dp}{\rho} + g\,dz = 0$$

The acceleration in the s direction is the time rate of change of the velocity in the s-direction. The velocity also depends on where the particle is along the stream line. Hence the velocity is a function of s and time t. This allows the acceleration component in the s-direction to be written as

$$a_s = \frac{dv(s,t)}{dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial s}\frac{\partial s}{\partial t} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial s}$$

limiting the derivation to steady state condition allows the s-direction acceleration to be written as

$$a_s = v\frac{\partial v}{\partial s}$$

~~allowing~~ applying Newton's second law in the s direction

$$V\,dv + \frac{1}{\rho}\,dp + g\,dz = 0$$