ADEGBITE TEMITOPE AHMED

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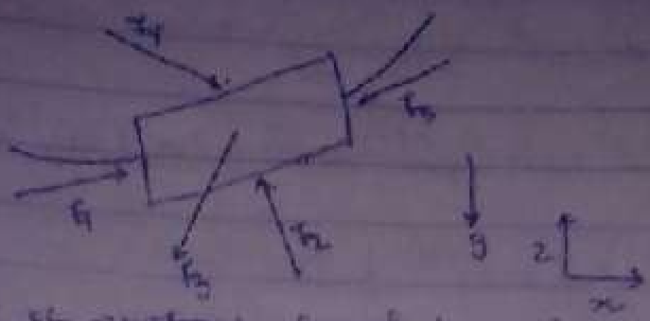
CHEMICAL ENGINEERING

FLUID FLOW

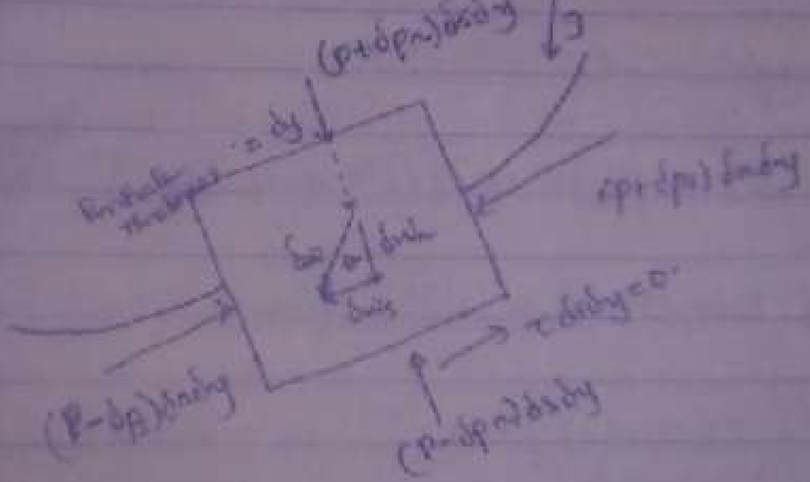


Component of newtons second law along stream line direction, s,

$$\sum \delta F_s = \delta m a_s = \delta m v \frac{\delta v}{\delta s} = \rho \delta \forall v \frac{\delta v}{\delta s}.$$

Component of weight force in the direction of streamline

$$\delta W_s = -\delta W \sin\theta = -\gamma \delta \forall \sin\theta$$



Take the 1st order taylor series expansion for the pressure field:

$$\delta p_s \approx \frac{\delta p}{\delta s} \cdot \frac{\delta s}{2}.$$

The net pressure force on the particle in the streamline direction

$$\delta F_{ps} = (p-\delta p_s)\delta n \delta y - (p+\delta p_s)\delta n \delta y = -2\delta p_s \delta n \delta y$$

$$= -\frac{\delta p}{\delta s} \delta s \delta n \delta y = -\frac{\delta p}{\delta s} \delta \forall$$

net force acting in the streamline direction on the particle is

$$\Sigma \delta F_s = \delta W_s + \delta F_{ps} = \left[-\gamma \sin\theta - \frac{\delta p}{\delta s}\right] \delta V$$

$$= -\gamma \sin\theta - \frac{\delta p}{\delta s} = \rho V \frac{\delta V}{\delta s} = \rho a_s$$

Noting that $\sin\theta = \frac{\delta z}{\delta s}$ and $V \frac{\delta V}{\delta s} = \frac{1}{2} \frac{\delta V^2}{\delta s}$, the above eqn can be rearranged and integrated:

$$-\gamma \frac{\delta z}{\delta s} - \frac{\delta p}{\delta s} = \frac{1}{2} \rho \frac{\delta V^2}{\delta s}$$

$$\delta p + \frac{1}{2} \rho d(V^2) + \gamma \delta z = 0$$

(along a stream line)

For constant acceleration of gravity

$$\int \frac{\delta p}{\rho} + \frac{1}{2} V^2 + gz = c$$

(along a stream line)

For steady, inviscid and incompressible ~~fluid~~ flow, we have the celebrated Bernoulli's Equation

$$\boxed{P + \frac{1}{2} \rho V^2 + \gamma z = c}$$

(along stream line)

$$P_1 + \frac{1}{2} \rho_1 V_1^2 + \gamma_1 z_1 = P_2 + \frac{1}{2} \rho_2 V_2^2 + \gamma_2 z_2$$