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Derivation of Bernoulli's equation from Newtons Second law.

The pressure acting on the element tending to acceleration fluid mass will be

$$P \cdot dA - (P + dP)\,dA = -dP \cdot dA \quad \text{---} \quad (1)$$

If y is the unit weight of fluid mass, gravity forces acting vertically downward will be $< y\,dA \cdot ds$. Component of gravity force along direction of motion will be

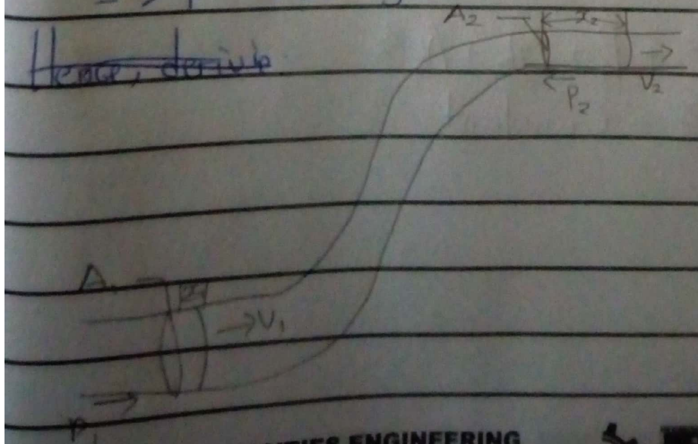
$$= -(y \cdot dA\,ds)\cos\theta = -(y \cdot dA \cdot ds)\frac{dz}{ds} = (y \cdot dA\,dz)$$

The velocity being V along the direction of fluid motion, acceleration for the steady flow will be $\left(V\frac{dv}{ds}\right)$. Applying Newtons law of motion $F = ma$

$$\therefore -(dP \times dA) - (y \cdot dA \cdot dz) = \frac{y}{g}\,dA \cdot ds \cdot \frac{Vdu}{ds}$$

$$= \frac{dP}{\rho} + Vdu + gdz = 0$$

~~Hence, derive~~



We examine a fluid section of mass m traveling to the right as shown above; Net work done in moving

$= W = W_1 + W_2 = F_1 x_1$ ---- ①

Where $P = F/A$ $\therefore F = PA$

Putting F in equ ①

$\Delta W = P_1 A_1 x - P_2 A_2 x_2$ ---- ②

The displaced fluid volume is the cross-sectional area A multiplied by the thickness x Where $V = A_1 x_1 = A_2 x_2$ ---- ③

Using equ 3 in ② whe have $\Delta W = (P_1 - P_2) V$ ---- ④

The energy change between the Initial and final positions

$\Delta E = E_2 - E_1 = (U_2 + K_2) - (U_1 + K_1)$

or $\Delta E = (mgh_2 + mv_2^2/2) - (mgh_1 + mv_1^2/2)$ ---- ⑤

Since $\Delta W = \Delta E$ ---- ⑥

Supstituting equ 4 and ⑤ in 6

$(P_1 - P_2) V = (mgh_2 + mv_2^2/2) - (mgh_1 + mv_1^2/2)$

Dividing equ 7 by the fluid volume

$P_1 - P_2 = (\rho g h_2 + \rho v_2^2/2) - (\rho g h_1 + \rho v_1^2/2)$

Where $\rho = m/v$

$\therefore P_1 + \rho g h_1 + \rho v_1^2/2 + P_2 + \rho g h_2 + \rho v_2^2/2$

$P + \rho g h + \rho v^2/2 = \text{Constant}$

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