NAME: GBENRO TIMOTHY

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QUESTION.

If $f(t) = 4t^2 - 2t + 5$ and $g(t) = 5t^3 - 6t^2 - 4t + 7$

a) $f(A)$ b) $g(A)$ c) $f(A) - g(A)$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 0 & 4 \\ 6 & 4 & 5 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 6 & -2 \\ -3 & 2 & 0 \\ 0 & 3 & -4 \end{pmatrix}$$

find Δt for A and B.

Solution.

(a) $f(t) = 4t^2 - 2t + 5$

$f(A) = 4A^2 - 2A + 5I$

$$f(A) = 4\begin{pmatrix} 1 & 2 & 3 \\ 3 & 0 & 4 \\ 6 & 4 & 5 \end{pmatrix}^2 - 2\begin{pmatrix} 1 & 2 & 3 \\ 3 & 0 & 4 \\ 6 & 4 & 5 \end{pmatrix} + 5\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 0 & 4 \\ 6 & 4 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ 3 & 0 & 4 \\ 6 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 1\times1+2\times3+3\times6 & 1\times2+2\times0+3\times4 & 1\times3+2\times4+3\times5 \\ 3\times1+0\times3+4\times6 & 3\times2+0\times0+4\times4 & 3\times3+0\times4+4\times5 \\ 6\times1+4\times3+5\times6 & 6\times2+4\times0+5\times4 & 6\times3+4\times4+5\times5 \end{pmatrix}$$

$$= \begin{pmatrix} 1+6+18 & 2+0+12 & 3+8+15 \\ 3+0+24 & 6+0+16 & 9+0+20 \\ 6+12+30 & 12+0+20 & 18+16+25 \end{pmatrix} = \begin{pmatrix} 25 & 14 & 26 \\ 27 & 22 & 29 \\ 48 & 32 & 59 \end{pmatrix}$$

$$f(A) = 4\begin{pmatrix} 25 & 14 & 26 \\ 27 & 22 & 29 \\ 48 & 32 & 59 \end{pmatrix} - 2\begin{pmatrix} 1 & 2 & 3 \\ 3 & 0 & 4 \\ 6 & 4 & 5 \end{pmatrix} + 5\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$f(A) = \begin{pmatrix} 100 & 56 & 104 \\ 108 & 88 & 116 \\ 192 & 128 & 236 \end{pmatrix} - \begin{pmatrix} 2 & 4 & 6 \\ 6 & 0 & 8 \\ 12 & 8 & 10 \end{pmatrix} + \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

$$f(A) = \begin{pmatrix} 103 & 52 & 98 \\ 102 & 93 & 108 \\ 180 & 120 & 231 \end{pmatrix}$$

(b) $g(t) = 5t^3 - 6t^2 - 4t + 7$

$g(A) = 5(A)^3 - 6(A)^2 - 4(A) + 7I$

$$g(A) = 5\begin{pmatrix} 1 & 2 & 3 \\ 3 & 0 & 4 \\ 6 & 4 & 5 \end{pmatrix}^3 - 6\begin{pmatrix} 1 & 2 & 3 \\ 3 & 0 & 4 \\ 6 & 4 & 5 \end{pmatrix}^2 - 4\begin{pmatrix} 1 & 2 & 3 \\ 3 & 0 & 4 \\ 6 & 4 & 5 \end{pmatrix} + 7\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$A^3 = A^2 . A$

$$= \begin{pmatrix} 25 & 14 & 26 \\ 27 & 22 & 29 \\ 48 & 32 & 59 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ 3 & 0 & 4 \\ 6 & 4 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 25\times1 + 14\times3 + 26\times6 & 25\times2 + 14\times0 + 26\times4 & 25\times3 + 14\times4 + 26\times5 \\ 27\times1 + 22\times3 + 29\times6 & 27\times2 + 22\times0 + 29\times4 & 27\times3 + 22\times4 + 29\times5 \\ 48\times1 + 32\times3 + 59\times6 & 48\times2 + 32\times0 + 59\times4 & 48\times3 + 32\times4 + 59\times5 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 223 & 154 & 261 \\ 267 & 170 & 314 \\ 498 & 332 & 567 \end{pmatrix}$$

$$g(A) = 5\begin{pmatrix} 223 & 154 & 261 \\ 267 & 170 & 314 \\ 498 & 332 & 567 \end{pmatrix} - 6\begin{pmatrix} 25 & 14 & 26 \\ 27 & 22 & 29 \\ 48 & 32 & 59 \end{pmatrix} - 4\begin{pmatrix} 1 & 2 & 3 \\ 3 & 0 & 4 \\ 6 & 4 & 5 \end{pmatrix} + 7\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$g(A) = \begin{pmatrix} 1115 & 770 & 1305 \\ 1335 & 850 & 1570 \\ 2490 & 1660 & 2835 \end{pmatrix} - \begin{pmatrix} 150 & 84 & 156 \\ 162 & 132 & 174 \\ 288 & 192 & 354 \end{pmatrix} - \begin{pmatrix} 4 & 8 & 12 \\ 12 & 0 & 14 \\ 24 & 16 & 20 \end{pmatrix} + \begin{pmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix}$$

$$g(A) = \begin{pmatrix} 968 & 678 & 1137 \\ 1161 & 725 & 1382 \\ 2178 & 1452 & 2468 \end{pmatrix}$$

(c) $f(A) - g(A)$

$$= \begin{pmatrix} 103 & 52 & 98 \\ 102 & 93 & 108 \\ 180 & 120 & 231 \end{pmatrix} - \begin{pmatrix} 968 & 678 & 1137 \\ 1161 & 725 & 1382 \\ 2178 & 1452 & 2468 \end{pmatrix}$$

$$= \begin{pmatrix} -865 & -626 & -1039 \\ -1059 & -632 & -1274 \\ -1998 & -1332 & -2237 \end{pmatrix}$$

(d) For A, Δt

$\Delta t = t^3 - tr(A)t^2 + (A_{11} + A_{22} + A_{33})t + \det A$

$tr(A) = 1 + 0 + 5 = 6$

$$\det A = \begin{vmatrix} \overset{+}{1} & \overset{-}{2} & \overset{+}{3} \\ 3 & 0 & 4 \\ 6 & 4 & 5 \end{vmatrix} = 1\begin{vmatrix} 0 & 4 \\ 4 & 5 \end{vmatrix} - 2\begin{vmatrix} 3 & 4 \\ 6 & 5 \end{vmatrix} + 3\begin{vmatrix} 3 & 0 \\ 6 & 4 \end{vmatrix}$$

$= 1(0-16) - 2(15-24) + 3(12-0)$

$= 1(-16) - 2(-9) + 3(12)$

$= -16 + 18 + 36$

$= 38$

$\Delta t = t^3 - 6t^2 + 6t + 38$

c. For B, Δt:

$\Delta t = t^3 + tr(B)t^2 + (B_{11} + B_{22} + B_{33})t + \det B$

$tr(B) = 1 + 2 + (-4) = -1$

$$\det(B) = \begin{vmatrix} 1 & 6 & 2 \\ -3 & 2 & 0 \\ 0 & 3 & -4 \end{vmatrix} = 1\begin{vmatrix} 2 & 0 \\ 3 & -4 \end{vmatrix} - 6\begin{vmatrix} -3 & 0 \\ 0 & -4 \end{vmatrix} + 2\begin{vmatrix} -3 & 2 \\ 0 & 3 \end{vmatrix}$$

$= 1(-8-0) - 6(12-0) + 2(-9-0)$

$= 1(-8) - 6(12) + 2(-9)$

$= -8 - 72 - 18$

$= -98$

$\Delta t = t^3 + (-1)t^2 + (-1)t + (-98)$

$\Delta t = t^3 - t^2 - t - 98$