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MATRIC NOS: 15/ENG06/025
DEPARTMENT: MECHANICAL ENGINEERING

Signature:

1- Use The Leibnitz Maclaurine Method to determine a series Solut for

$$\underbrace{(1-x^2)\frac{d^2y}{dx^2}}_{\text{Sub 1}} - \underbrace{2x\frac{dy}{dx}}_{\text{Sub 2}} + \underbrace{2y}_{\text{Sub 3}} = 0$$

~~For sub 1~~ $(1-x^2)y'' - 2xy' + 2y = 0$

for Sub 1

$= (1-x^2)y^{n+2} - 2xy^{n+1} - 2ny^n$

$\Rightarrow (1-x^2)y^{n+2} - 2xy^{n+1} - 2ny^n$

for Sub 2

$\Rightarrow -2xy^{n+1} - 2ny^n$

Sub 3

$\Rightarrow 2y^n$

Combining Sub 1, 2 and 3

$(1-x^2)y^{n+2} - 2xy^{n+1} - 2ny^n - 2xy^{n+1} - 2ny^n + 2y^n = 0$

Setting $x=0$

$y^{n+2} - 2ny^n - 2ny^n + 2y^n = 0$

~~$y^{n+2} = 2y^n(n-n$~~

$y^{n+2} = 2ny^n + 2ny^n - 2y^n$

~~If n=0~~ $y^{n+2} = 2y^n(n+n-1]$

~~n=0~~ $n=0$

$y^{0+2} = 2y^0[0+0-1]$

$(y^2)_0 = 2(y^0)_0$

$n=1$

$(y^3)_0 = 2y'(1+1-1]$

$(y^3)_0 = 2y'[1]$

$(y^3)_0 = 2(y')_0$

$n=2$

$(y^4)_0 = 2y^2(2+2-1]$

$(y^4)_0 = 2y^2[3]$

$(y^4)_0 = 6y^2 = 6[2(y^0)_0]$

$= 12(y^0)_0$

$n = 3$

$(y^5)_0 = 2(y^3)_0(3+3-1)$

$(y^5)_0 = 2(y^3)_0[5]$

$(y^5)_0 = 10(y^3)_0 \Rightarrow 10[2(y')_0]$

$= 20(y')_0$

$n = 4$

$(y^6)_0 = 2(y^4)_0[4+4-1]$

$= 2(y^4)_0[7]$

$= 14(y^4)_0 \Rightarrow 14[12(y^0)_0]$

$= 168(y^0)_0$

$n = 5$

$(y^7)_0 = 2(y^5)_0[5+5-1]$

$= 2(y^5)_0[9]$

$= 18(y^5)_0 \Rightarrow 18[20(y')_0]$

$= 360(y')_0$

$y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(y'')_0 + \frac{x^3}{3!}(y''')_0 + \frac{x^4}{4!}(y^{iv})_0$

$y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(2y^0)_0 + \frac{x^3}{3!}(2y')_0 + \frac{x^4}{4!}(12y^0)_0 + \frac{x^5}{5!}(20y')_0 + \frac{x^6}{6!}(168y^0)_0$
$+ \frac{x^7}{7!}(360y')_0$

2. i) $3e^{-4t} - 5e^{4t}$

~~1~~ Using $\frac{1}{s-a}$

$3 \cdot \frac{1}{s+4} - 5 \cdot \frac{1}{s-4}$

$\Rightarrow \frac{3}{s+4} - \frac{5}{s-4}$

ii) $\sin 4t + \cos 4t$

Using $\sin at = \frac{a}{s^2+a^2}$

$$\cos at = \frac{s}{s^2+a}$$

$$\therefore \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$$

$$\Rightarrow \frac{4}{s^2+16} + \frac{s}{s^2+16}$$

iii) $t^3 + 2t^2 - t + 4$

$$\Rightarrow \frac{3!}{s^4} + 2 \cdot \frac{2!}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$\Rightarrow \frac{6}{s^4} + 2 \cdot \frac{2}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$\frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

iv) $e^{-2t} \cos 5t$

Using $\cos at = \dfrac{s}{s^2+a^2}$

$$\cos 5t = \frac{s}{s^2+25}$$

Replace s by the shift of e^{-2t} by $s+2$

$$= \frac{[s+2]}{[s+2]^2+25}$$

$$= \frac{[s+2]}{[s(s+2)+2(s+2)]+25}$$

$$= \frac{s+2}{[s^2+2s+2s+4]+25} = \frac{s+2}{s^2+4s+29}$$

v) $t \sin 5t$

$F = -\frac{dF}{ds}$ ~~Usin~~

$\sin at = \frac{a}{s^2+a^2}$

$= \frac{5}{s^2+25} \Rightarrow \frac{u}{v}$

Using Product Rule

$$\frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2}$$

$U = 5$ $\qquad \frac{du}{dx} = 0$

$V = s^2+25$ $\qquad \frac{dv}{dx} = 2s$

$$= \frac{[s^2+25]0 - 5[2s]}{[s^2+25]^2}$$

$$= \frac{-10s}{[s^2+25]^2}$$

$$\therefore = -\frac{[-10s]}{[s^2+25]^2}$$

$$= \frac{10s}{[s^2+25]^2}$$

vi) $\frac{e^{-t} - e^{-2t}}{t}$

$$= \ln\sqrt{\frac{s-1}{1} - \frac{s+2}{1}}$$

$V = s^2+25$ $\frac{dv}{dx} =$

$= \frac{[s^2+25]0 - 5[2s]}{[s^2+25]^2}$

vii) $e^{4t}\cos 2t$

$\cos at = \frac{s}{s^2+a^2}$

$\cos 2t = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$

Replace s ~~by~~ shift of e^{4t} by $s-4$

$= \frac{[s-4]}{[s-4]^2+4}$

$= \frac{[s-4]}{[s(s-4)-4(s-4)]+4}$

$= \frac{[s-4]}{s^2-4s-4s+16+4}$

$= \frac{[s-4]}{s^2-8s+20}$

(viii) $t\sin 2t$

Using $\sin at = \frac{a}{s^2+a^2}$ note $f = -\frac{d}{ds}$

$\sin 2t = \frac{2}{s^2+4}$ $\frac{u}{v}$

Using Product Rule

$\frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2}$

$u = 2$ $\frac{du}{dx} = 0$

$v = s^2+4$ $\frac{dv}{dx} = 2s$

$\frac{[s^2+4]0 - 2[2s]}{[s^2+4]^2}$

$\frac{dx}{} \quad \frac{}{dx}$

v^2

$U = 5$ $\quad \frac{du}{dx} = 0$

$V = s^2 + 25$ $\quad \frac{dv}{dx} = 2s$

$= \frac{[s^2+25]0 - 5[2s]}{[s^2+25]^2}$

$= -10s$

xi)

$= \frac{-2[2s]}{[s^2+4]^2}$

$= \frac{-[-2][2s]}{[s^2+4]^2}$

$= \frac{4s}{[s^2+4]^2}$

(ix) $t^3 + 4t^2 + 5$

$\Rightarrow \frac{3!}{s^4} + 4 \cdot \frac{2!}{s^3} + \frac{5}{s}$

$\Rightarrow \frac{6}{s^4} + 4 \cdot \frac{2}{s^3} + \frac{5}{s}$

$\Rightarrow \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

$\frac{1}{s^4}[6 + 8s + 5s^3]$

(x) $e^{3t}(t^2+4)$

$\cancel{t^2+4}$ $L(t^2+4]$

$= \frac{2!}{s^3} + \frac{4}{s}$

$= \frac{2}{s^3} + \frac{4}{s}$

Replace s by ~~e^{3t}~~ of shift e^{3t} ~~of~~ by $(s-3)$

$= \frac{1}{s^3}[2 + 4s^2]$

$= \frac{1}{[s-3]^3}[2 + 4[s-3]^2]$

$A = \frac{-24}{-12} = 2$

$\frac{dy}{dx} = 2s$

$[2s]$

xi) $t^2 \cos t$

$\cos t = \frac{s}{s^2+1}$ $\frac{u}{v}$

$f = -\frac{d}{ds}$

Using product Rule

$$\frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2}$$

$U = s$ $du = 0$

$V = s^2+1$ $dv = 2s$

$$= \frac{[s^2+1]0 - 2s[s]}{[s^2+1]^2}$$

$$= -\frac{(-2s^2)}{[s^2+1]^2}$$

$$= \frac{2s^2}{[s^2+1]^2} \quad \frac{u}{v}$$

for the second derivative

$U = 2s^2$ $\frac{du}{dx} = 4s$

$V = [s^2+1]^2$ $dv = 4s^3+4s$

Using $\frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2}$

$$\Rightarrow \frac{[s^2+1]^2 4s - 2s^2[4s^3+4s]}{[s^2+1]^4}$$

$$\Rightarrow \frac{[s^2+1]4s - 2s^2[4s^3+4s]}{[s^2+1]^3}$$

$$\Rightarrow \frac{4s^3+4s - \cancel{8} 8s^5+8s^3}{[s^2+1]^3}$$

$$\Rightarrow -\left\{\frac{12s^3+4s-8s^5}{[s^2+1]^3}\right\} = \frac{12s^3-4s+8s^5}{[s^2+1]^3}$$

$$v^2$$

$u = s$ $\quad \frac{du}{dx} = 0$

$v = s^2+25$ $\quad \frac{dv}{dx} = 2s$

$$= \frac{[s^2+25]0 - s[2s]}{[s^2+25]^2}$$

$$= \frac{-10s}{[s^2+25]^2}$$

xii $\frac{\sinh 2t}{t}$

Ans $= \ln \sqrt{\frac{s^2-4}{2}}$

3. $$\frac{s-5}{(s-3)(s-4)} = \frac{A}{(s-3)} + \frac{B}{(s-4)}$$

3.)

$A(s-4) + B(s-3)$

$As - 4A + Bs - 3B$

$\therefore \; A - 4A - 3B = -5 \quad - ① \times 1$

$A + B = 1 \quad - ② \times -4$

$-4A - 3B = -5$

$+ \; -4A - 4B = -4$

$B = -1$

Substitut $B = -1$ into ①

$-4A - 3(-1) = -5$

$-4A + 3 = -5$

$-4A = -5 - 3$

$\frac{\cancel{-4}A}{\cancel{-4}} = \frac{-8}{-4}$

$A = 2$

$$= \frac{2}{(s-3)} + \frac{-1}{(s-4)}$$

$\Rightarrow \; 2e^{3t} + e^{4t}$

-24

$A = \frac{-24}{-12} = 2$

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ii) $\dfrac{2s-6}{(s-2)(s-4)} = \dfrac{A}{(s-2)} + \dfrac{B}{(s-4)}$

$= A(s-4) + B(s-2)$

$= As - 4A + Bs - 2B$

$-4A - 2B = -6 \quad — ① \times 1$

$A + B = 2 \quad — ② \times -4$

$-4A - 2B = -6 \quad — ①$

$-4A - 4B = -8 \quad — ②$

Substrac ① - ②

$\dfrac{\cancel{2}B}{\cancel{2}} = \dfrac{2}{2}$

$B = 1$

Substitut $B = 1$ ①

$-4A - 2(1) = -6$

$-4A - 2 = -6$

$-4A = -6 + 2$

$\dfrac{\cancel{-4}A}{\cancel{-4}} = \dfrac{-4}{-4}$

$A = 1$

$= \dfrac{1}{s-2} + \dfrac{1}{s-4}$

$= e^{2t} + e^{4t}$

$(s-4)$ s $(s-4)$

$A(s-4) + Bs$

$As - 4A + Bs$

$A + B = 5$ — ①

$-4A = -8$ — ②

$\frac{-4A}{-4} = \frac{-8}{-4}$

$A = 2$

Subotitut $A = 2$ into ①

$2 + B = 5$

$B = 5 - 2$

$B = \underline{3}$

$\therefore \frac{2}{s} + \frac{3}{s-4}$

$2t + 3e^{4t}$