QUESTION 1.

$(1-x^2)\, d^2y/dx^2 - 2x\, dy/dx + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0.$

SUB 1

$u = y''$ $\quad u^n = y^{(n+2)}$

$v = 1-x^2$ $\quad v' = -2x$ $\quad v'' = -2$

$y^{(n)} = y^{(n+2)} \cdot 1-x^2 + ny^{(n+1)} \cdot -2x + \frac{n(n-1)}{2} y^n \cdot -2$

$y^{(n)} = (1-x^2)y^{(n+2)} - 2nxy^{(n+1)} - n(n-1)y^n$

SUB 2

$u = y'$ $\quad u^n = y^{(n+1)}$

$v = -2x$ $\quad v' = -2$

$y^{(n)} = -2x\, y^{(n+1)} + ny^{(n)} \cdot -2$

$= -2xy^{(n+1)} - 2ny^{(n)}.$

Q i. $L\left[3e^{-4t} - 5e^{4t}\right]$

$$\frac{3}{s+4} - \frac{5}{s-4} = \frac{3(s-4) - 5(s+4)}{(s+4)(s-4)}$$

$$= \frac{3s - 12 - 5s - 20}{\cancel{s+4}\ s^2 - 16} = \frac{-2s - 32}{s^2 - 16}$$

ii. $L\left[\sin 4t + \cos 4t\right]$

$$\frac{4}{s^2+4^2} + \frac{s}{s^2+4^2} = \frac{4+s}{s^2+16}$$

iii. $L\left[t^3 + 2t^2 - t + 4\right]$

$$= \frac{3!}{s^4} + \frac{2 \cdot 2!}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s} = \frac{6 + 4s - s^2 + 4s^3}{s^4}$$

iv. $L[e^{-2t}\cos 5t]$

$\cos 5t = \frac{s}{s^2+5^2}$

let $s = s+2$

$L[e^{-2t}\cos 5t] = \frac{s+2}{(s+2)^2+5^2} = \frac{s+2}{s^2+\cancel{4s}+29}$

v. $L[t\sin 3t]$

$L[\sin 3t] = \frac{3}{s^2+3^2} = \frac{3}{s^2+9}$

$-F'(s) = -\frac{d}{ds}\left[\frac{3}{s^2+9}\right]$

$u = 3$ $\quad du/ds = 0$

$v = s^2+9$ $\quad dv/ds = 2s$

Using quotient rule,

$\frac{v\, du/ds - u\, dv/ds}{v^2}$

$= \frac{(s^2+9)(0) - 3(2s)}{(s^2+9)^2} = \frac{0-6s}{(s^2+9)^2} = \frac{-6s}{(s^2+9)^2}$

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vii. $L[e^{4t}\cos 2t]$

$L[\cos 2t] = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$

Let $s = s-4$

$L[e^{4t}\cos 2t] = \frac{s-4}{(s-4)^2+4} = \frac{s-4}{s^2-8s-12}$

viii. $L[t\sin 2t]$

$L[\sin 2t] = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$

$L[t\sin 2t] = -f'(s) = -\frac{d}{ds}\left[\frac{2}{s^2+4}\right]$

$u = 2$ $\qquad du/ds = 0$

$v = s^2+4$ $\qquad dv/ds = 2s$.

Using quotient rule

$-\left[\frac{v\,du/ds - u\,dv/ds}{v^2}\right]$

$= -\left[\frac{s^2+4(0) - 2(2s)}{(s^2+4)^2}\right] = \frac{4s}{(s^2+4)^2}$.

ix. t^3+4t^2+5

$= \frac{3!}{s^4} + \frac{4.2!}{s^3} + \frac{5}{s}$

$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s} = \frac{6+8s+5s^3}{s^4}$

x. $L[e^{3t}(t^2+4)]$

$L[t^2e^{3t}] + L[4e^{3t}]$.

$L[e^{3t}] = \frac{1}{s-3}$

$L[t^2] = \frac{2!}{s^{2+1}} = \frac{2}{s^3}$

$s = s-3$

$L[t^2e^{3t}] = \frac{2}{(s-3)^3}$.

$L[e^{3t}(t^2+4)] = \frac{2}{(s-3)^3} + \frac{4}{s-3}$

$= \frac{2+4(s-3)^2}{(s-3)^3} = \frac{2+4s^2-24s+36}{(s-3)^3}$

$= \frac{4s^2-24s+38}{(s-3)^3}$.

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xi. $t^2 \cos t$

$$L[\cos t] = \frac{s}{s^2+1^2} = \frac{s}{s^2+1}$$

$$-f'(s) = \frac{-d}{ds}\left[\frac{s}{s^2+1}\right]$$

$u = s \qquad du/ds = 1$

$v = s^2+1 \qquad dv/ds = 2s$

$$\frac{v\,du/ds - u\,dv/ds}{v^2}$$

$$-\left(\frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2}\right) = \left(\frac{s^2+1-2s^2}{(s^2+1)^2}\right)$$

$$L^{-1}[t\cos t] = \frac{-s^2+1}{(s^2+1)^2} = \frac{-1\cancel{(s^2+1)}}{(s^2+1)^4} = \frac{-1}{s^2+1}$$

$$= 1/s^2+1$$

~~Ans.~~ ~~sin 3t~~ $L[t^2 \cos t] = \frac{-d}{ds}\left[\frac{1}{s^2+1}\right]$

$u = 1 \qquad du/ds = 0$

$v = s^2+1 \qquad dv/ds = 2s$

~~$v\,du/ds$~~ $$\frac{(s^2+1)(0) - (1)(2s)}{(s^2+1)^2} = \frac{-2s}{(s^2+1)^2}$$

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QUESTION 3.

e. $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$A(s-4) + B(s-3) = s-5$

at $s = 4$

$B(1) = 4-5 = -1$

$B = -1$

at $s = 3$

$A(-1) + 0 = -2$

$-A = -2$

$A = 2$

$= \frac{2}{s-3} - \frac{1}{s-4}$

$\Rightarrow 2e^{3t} - e^{4t}$

$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$A(s-4) + B(s-2) = 2s-6$

at $s = 4$

$0 + B(2) = 2(4) - 6 = 2$

$B = \frac{2}{2} = 1$

at $s = 2$

$A(2-4) + 0 = 2(2) - 6 = 4 - 6 = -2$

$-2A = -2$

$A = 1$

$\Rightarrow \frac{1}{s-2} + \frac{1}{s-4}$

$\Rightarrow e^{2t} + e^{4t}$

iii. $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$A(s-4) + Bs = 5s - 8$

at $s = 0$

$A(-4) + 0 = 5(0) - 8 = -8$

$-4A = -8$

$A = 2$

at $s = 4$

$0 + 4B = 5(4) - 8 = 12$

$4B = 12$

$B = 3$

$\Rightarrow \frac{2}{s} + \frac{3}{s-4}$

$= 2 + 3e^{4t}$

iv. $\frac{s^2 - 3s - 4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$

$A(s-1)^2 + B(s-3)(s-1) + C(s-3) = s^2 - 3s - 4$

at $s = 1$

$0 + 0 + C(-2) = 1^2 - 3(1) - 4 = 1 - 7 = -6$

$-2C = -6$

$C = 3$

at $s = 3$

$A(2)^2 + 0 + 0 = 3^2 - 3(3) - 4 = 9 - 9 - 4 = -4$

$4A = -4$

$A = -1$

$A + B = 1$

$B = 1 - A = 1 - (-1) = 1 + 1 = 2$

$\Rightarrow \frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}$

$\Rightarrow -e^{3t} + 2e^{t} + 3te^{t}$.

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$$L^{-1}\left[\frac{s-5}{s^2+4s+20}\right] = L^{-1}\left[\frac{s-5}{(s+2)^2+4^2}\right]$$

$$= L^{-1}\left[\frac{s+2}{(s+2)^2+4^2}\right] + L^{-1}\left[\frac{-7}{(s+2)^2+4^2}\right]$$

$$= e^{-2t} \cos 4t - 7/4 \sin 4t$$