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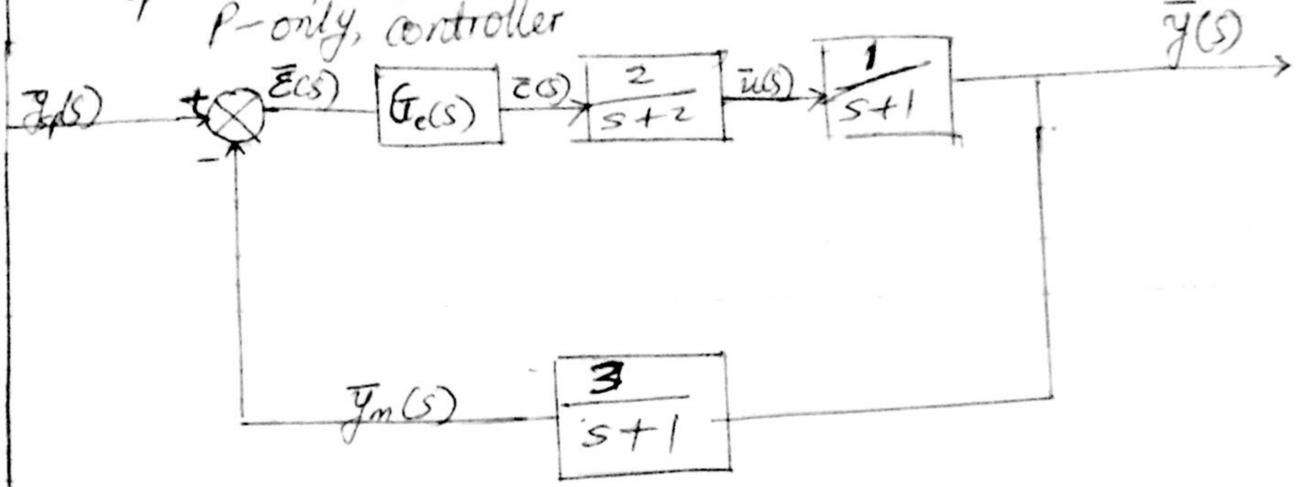
Assignment IV

Question 1 Answer (Routh stability approach)

$\tau_p = 1$, $\tau_f = \frac{1}{2}$, $\tau_m = \frac{1}{3}$, ~~and $\tau_1 = 0.25$~~

P-only, controller



from the characteristic eqn

$$1 + G_f G_m G_c G_p = 0$$

but $G_f = \dfrac{1}{\tau_f s+1} = \dfrac{1}{\frac{1}{2}s+1} = \dfrac{1}{\frac{1}{2}(s+2)} = \dfrac{2}{s+2}$

$$G_m = \frac{1}{\tau_m s+1} = \frac{1}{\frac{1}{3}s+1} = \frac{1}{\frac{1}{3}(s+3)} = \frac{3}{s+3}$$

$$G_p = \frac{1}{\tau_p s+1} = \frac{1}{s+1} \quad ; \quad G_c = K_c$$

$$\therefore 1 + G_f G_m G_c G_p = 1 + \left(\frac{2}{s+2}\right)\left(\frac{3}{s+3}\right)\left(\frac{K_c}{1}\right)\left(\frac{1}{s+1}\right) = 0$$

$$= 1 + \frac{\cancel{6K_c}\, 6K_c}{(s+2)(s+3)(s+1)} = 0$$

$$1 + G_f G_m G_c G_p = \frac{(s+2)(s+3)(s+1) + 6K_c}{(s+2)(s+3)(s+1)} = 0$$

$$\therefore (s+2)(s+3)(s+1) + 6K_c = 0$$

$$s^3 + 6s^2 + 11s + 6 + 6K_c = 0$$

$$s^3 + 6s^2 + 11s + 6 + 6K_c = 0$$

All coefficients are positive,
$\therefore$ the routh array is

$$\begin{array}{ll} 1 & 11 \\ 6 & 6+6K_c \\ b_1 & b_2 \\ c_1 & \end{array}$$

Where $b_1 = \frac{6(11) - 1(6+6K_c)}{6} = \frac{66 - 6 - 6K_c}{6}$

$$b_1 = \frac{60 - 6K_c}{6} = \frac{60}{6} - \frac{\cancel{6}K_c}{\cancel{6}}$$

$$b_1 = 10 - K_c$$

$b_2 = 0$,

$$c_1 = \frac{b_1(6+6K_c) - 6(b_2)}{b_1} = \frac{b_1(6+6K_c) - 6(0)}{b_1}$$

$$c_1 = \frac{b_1(6+6K_c) - 0}{b_1}$$

$$c_1 = \frac{\cancel{b_1}(6+6K_c)}{\cancel{b_1}}$$

$$c_1 = 6+6K_c$$

$\therefore$ To have a stable system,
Considering ~~b~~ b_1, the condition for b_1 which is that

$$10 - K_c > 0$$

$$\therefore -K_c > -10$$

$$\Rightarrow K_c < 10$$

Considering c_1, the condition for c_1, which is that

$$6 + 6K_c > 0$$

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$6K_c > -6$

$\therefore K_c > \frac{-6}{6}$

$\therefore K_c > -1$

$\therefore$ finally the values of K_c for which the control system falls in the range below

$$\boxed{-1 < K_c < 10}$$

$\therefore$ In the ~~set form~~ matrix form, K has values of $K_c = [0, 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9]$

Question 2 [Routh ~~Array~~ stability Approach]

PI controller: $\tau_p = 1$, $\tau_f = \frac{1}{2}$, $\tau_m = \frac{1}{3}$, $K_c = 5$, $\tau_I = 0.25$

Using the characteristic equation, $G_c = K_c + \frac{K_c}{\tau_I s}$

$$1 + G_f G_m G_c G_p = 1 + \left(\frac{2}{s+2}\right)\left(\frac{3}{s+3}\right)\left(5 + \frac{5}{0.25s}\right)\left(\frac{1}{s+1}\right) = 0$$

$$1 + G_f G_m G_c G_p = 1 + \left(\frac{2}{s+2}\right)\left(\frac{3}{s+3}\right)\left(5 + \frac{20}{s}\right)\left(\frac{1}{s+1}\right) = 0$$

$$1 + G_f G_m G_c G_p = 1 + \left(\frac{2}{s+2}\right)\left(\frac{3}{s+3}\right)\left(\frac{5s+20}{s}\right)\left(\frac{1}{s+1}\right) = 0$$

$$= 1 + \frac{6(5s+20)}{(s+2)(s+3)(s)(s+1)} = 0$$

$$= 1 + \frac{30s+120}{(s+2)(s+3)(s)(s+1)} = 0$$

$$= \frac{(s+2)(s+3)(s)(s+1) + 30s + 120}{(s+2)(s+3)(s)(s+1)} = 0$$

$$= (s+2)(s+3)(s)(s+1) + 30s + 120 = 0$$

Expanding, we have

$\Rightarrow s^4 + 6s^3 + 11s^2 + 36s + 120 = 0$

Using the routh array, we have

$a_0 = 1, \quad a_1 = 6, \quad a_2 = 11, \quad a_3 = 36, \quad a_4 = 120$

Row 1 ÷ $a_0 \quad a_2 \quad a_4 \quad a_6 \; - - -$

Row 2 ÷ $a_1 \quad a_3 \quad a_5 \quad a_7 \; - - -$

Row 3 $A_1, \quad A_2 \quad A_3$

Row 4 $B_1, \quad B_2, \quad B_3$

Row 5 $C_1, \quad C_2, \quad C_3$

The condition for stability is that all values in the first left colum must be +ve

$\therefore \begin{bmatrix} a_0 \\ a_1 \\ A_1 \\ B_1 \\ C_1 \end{bmatrix}$ must be +ve (positive)

The calculations of Row 3, 4 & 5 are shown in the next page

∴ Row 1 ÷ 1 11 120

Row 2 ÷ 6 36 0

Row 3 ÷ 5 120 0

Row 4 −108 0 0

Row 5 120 0

Conclusion

from the ~~observation~~ table above, it is observed that not all the values in the left column are positive in the array, ∴ the system is <u>unstable</u>. ← Answer

Appendix

but to get Row 3, A_1 in

$$A_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$A_1 = \frac{6(11) - 1(36)}{6} = \frac{66-36}{6} = \frac{\cancel{30}^{5}}{\cancel{6}_1}$$

$$A_1 = 5$$

to get A_2,

$$A_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} = \frac{6(120) - 1(0)}{6}$$

$$A_2 = \frac{\cancel{6}(120)}{\cancel{6}} = 120$$

to get A_3,

$$A_3 = \frac{a_1 a_6 - a_0 a_7}{a_1} = \frac{\cancel{6}(0) - 0}{6} = 0$$

$$B_1 = \frac{A_1 a_3 - a_1 A_2}{A_1} = \frac{5(36) - 6(120)}{5} = \frac{-540}{5}$$

$$B_1 = -108$$

$$B_2 = \frac{A_1 a_5 - a_1 A_3}{A_1} = \frac{5(0) - 6(0)}{5} = 0$$

$$B_3 = 0, \quad C_1 = \frac{B_1 A_2 - A_1 B_2}{B_1} = \frac{-108(120) - 5(0)}{-108}$$

$$C_1 = \frac{\cancel{-108}(120)}{\cancel{-108}}$$

$$\therefore C_1 = +120$$

$$C_2 = \frac{B_1 A_3 - A_1 B_3}{B_1} = \frac{-108(0) - 5(0)}{-108} = 0$$