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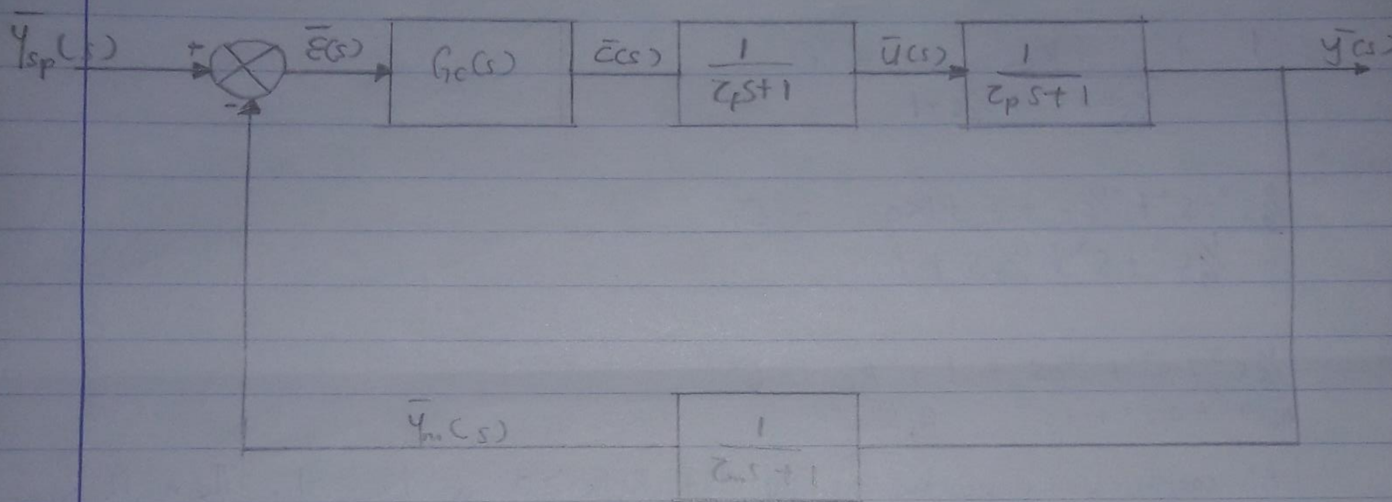
14/ENG07/016
CHE 531
ASSIGNMENT 4



P-only controller: $G_c(s) = K_c$

$\tau_p = 1 \quad \tau_f = 1/2 \quad \tau_m = 1/3$

$$G_f = \frac{1}{\tau_f s+1} = \frac{1}{\frac{1}{2}s+1}$$

$$G_p = \frac{1}{\tau_p s+1} = \frac{1}{s+1}$$

$$G_m = \frac{1}{\tau_m s+1} = \frac{1}{\frac{1}{3}s+1}$$

Characteristic equation applying Routh's stability approach: $1 + G_p G_f G_c G_m = 0$

$$1 + G_p G_f G_c G_m = 1 + \left(\frac{1}{s+1}\right)\left(\frac{1}{\frac{1}{2}s+1}\right) \cdot K_c \cdot \left(\frac{1}{\frac{1}{3}s+1}\right) = 0$$

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$$1 + \frac{K_c}{(s+1)(\frac{1}{2}s+1)(\frac{1}{3}s+1)} = 0$$

$$(s+1)(\tfrac{1}{2}s+1)(\tfrac{1}{3}s+1) = (\tfrac{1}{2}s^2 + s + \tfrac{1}{2}s + 1)(\tfrac{1}{3}s+1)$$
$$= (\tfrac{1}{2}s^2 + \tfrac{3}{2}s + 1)(\tfrac{1}{3}s+1)$$
$$= \tfrac{1}{6}s^3 + \tfrac{1}{2}s^2 + \tfrac{1}{3}s + \tfrac{1}{2}s^2 + \tfrac{3}{2}s + 1$$
$$= \tfrac{1}{6}s^3 + s^2 + \tfrac{11}{6}s + 1$$

$$1 + \frac{K_c}{\frac{1}{6}s^3 + s^2 + \frac{11}{6}s + 1} = 0$$

$$\frac{\frac{1}{6}s^3 + s^2 + \frac{11}{6}s + 1 + K_c}{\frac{1}{6}s^3 + s^2 + \frac{11}{6}s + 1} = 0$$

$$\tfrac{1}{6}s^3 + s^2 + \tfrac{11}{6}s + 1 + K_c = 0$$
$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} \quad a_3 s^{n-3}$$

All coefficient are positive provided that $1 + K_c > 0$ or $K_c > -1$. The routh array is

Row 1	$\frac{1}{6}$ a_0	$\frac{11}{6}$ a_2
Row 2	1 a_1	$1+K_c$ a_3
Row 3	A_1	A_2
Row 4	B_1	B_2

$$A_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} = \frac{1 \times \frac{11}{6} - \frac{1}{6}(1+K_c)}{\cancel{\frac{1}{6}}} = \frac{\frac{11}{6} - \frac{1}{6} - \frac{1}{6}K_c}{\cancel{\frac{1}{6}}} = \frac{\frac{5}{3} - \frac{1}{6}K_c}{\cancel{\frac{1}{6}}}$$

$$A_1 = \tfrac{5}{3} - \tfrac{1}{6}K_c$$

$$A_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} = \frac{1 \times 0 - \frac{1}{6} \times 0}{1} = 0$$

$$B_1 = \frac{A_1 a_3 - A_2 a_1}{A_1} = \frac{(\frac{5}{3} - \frac{1}{6}K_c)(1+K_c) - 0 \times 1}{\frac{5}{3} - \frac{1}{6}K_c} = \frac{(\frac{5}{3} - \frac{1}{6}K_c)(1+K_c)}{(\frac{5}{3} - \frac{1}{6}K_c)}$$

$B_1 = 1 + K_c$

To have a stable system, each element in the left column of the Routh array must be positive. Element A_1 will be positive if $K_c > (5/3 \div 1/6) = 10$
Similarly B_1 will be positive if $K_c > -1$.
Thus, it can be concluded that the system will be stable if $-1 < K_c \leq 10$

② PI Controller: $G_c(s) = K_c + \frac{K_c}{\tau_I S} = 5 + \frac{5}{0.25S} = \frac{5/4 S + 5}{0.25S}$

$G_f = \frac{1}{1/2 S + 1}$ $\quad G_p = \frac{1}{S+1}$ $\quad G_m = \frac{1}{1/3 S + 1}$

$1 + G_p G_f G_c G_m = 0$

$$1 + \left(\frac{1}{S+1}\right)\left(\frac{1}{1/2 S+1}\right)\left(\frac{5/4 S + 5}{1/4 S}\right)\left(\frac{1}{1/3 S+1}\right) = 0$$

$$1 + \frac{5/4 S + 5}{(S+1)(1/2 S+1)(1/4 S)(1/3 S+1)} = 0$$

from question 1 $(S+1)(1/2 S+1)(1/3 S+1) = 1/6 S^3 + S^2 + 11/6 S + 1$
$(S+1)(1/2 S+1)(1/3 S+1)(1/4 S) = (1/6 S^3 + S^2 + 11/6 S + 1)(1/4 S)$
$= 1/24 S^4 + 1/4 S^3 + 11/24 S^2 + 1/4 S$

$$1 + \frac{5/4 S + 5}{1/24 S^4 + 1/4 S^3 + 11/24 S^2 + 1/4 S} = 0$$

$$\frac{1/24 S^4 + 1/4 S^3 + 11/24 S^2 + 1/4 S + 5/4 S + 5}{1/24 S^4 + 1/4 S^3 + 11/24 S^2 + 1/4 S} = 0$$

$$\frac{1/24 S^4 + 1/4 S^3 + 11/24 S^2 + 3/2 S + 5}{1/24 S^4 + 1/4 S^3 + 11/24 S^2 + 1/4 S} = 0$$

The characteristic equation is $1/24 S^4 + 1/4 S^3 + 11/24 S^2 + 3/2 S + 5 = 0$

$a_0 S^n + a_1 S^{n-1} + a_2 S^{n-2} + a_3 S^{n-3}\ a_4 S^{n-4} = 0$

Row 1	$\frac{1}{24}$	$\frac{11}{24}$	5
Row 2	$\frac{1}{4}$	$\frac{3}{2}$	0
Row 3	$\frac{5}{24}$	5	
Row 4	$-\frac{9}{2}$		
Row 5	5		

$$A_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} = \frac{\frac{1}{4} \times \frac{11}{24} - \frac{1}{24} \times \frac{3}{2}}{\frac{1}{4}} = \frac{5}{24}$$

$$A_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} = \frac{\frac{1}{4} \times 5 - \frac{1}{24} \times 0}{\frac{1}{4}} = 5$$

$$A_3 = 0$$

$$B_1 = \frac{A_1 a_3 - A_2 a_1}{A_1} = \frac{\frac{5}{24} \times \frac{3}{2} - 5 \times \frac{1}{4}}{\frac{5}{24}} = -\frac{9}{2}$$

$$B_2 = \frac{A_1 a_5 - a_1 A_3}{A_1} = 0$$

$$C_1 = \frac{B_1 A_2 - A_1 B_2}{B_1} = \frac{-\frac{9}{2} \times 5 - \frac{5}{24} \times 0}{-\frac{9}{2}} = 5$$

All the coefficient in the first column are not positive therefore the system is unstable.