ENG 361
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15/ENG04/009
Elect / Elect

1) $y = e^{x^2+x}$

show that

$y'' = y'(2x+1) + 2y$ and prove

$y^{(n+2)} = (2x+1)\,y^{(n+1)} + 2(n+1)\,y^{(n)}$

$y = e^{x^2+x}$ ——— 1

$y' = (2x+1)\,e^{x^2+x}$ ——— 2

using productrule

$u = 2x+1$

$du = 2$

$v = e^{x^2+x}$

$dv = (2x+1)\,e^{x^2+x}$

$y'' = u\frac{dv}{dx} + v\frac{du}{dx}$

$= (2x+1)(2x+1)\,e^{x^2+x} + 2e^{x^2+x}$

from eqn 1 & 2

$y'' = y'(2x+1) + 2y$

$y^{(2)} = y^{(1)}(2x+1) + 2y$

$y^{(n+2)} = (2x+1)\,y^{(n+1)} + 2ny^{(n)} + 2y^{(n)}$

$y^{(n+2)} = (2x+1)\,y^{(n+1)} + (2n+2)\,y^{(n)}$

$y^{(n+2)} = (2x+1)\,y^{(n+1)} + 2(n+1)\,y^{(n)}$

2) i $y = x^3 e^{4x}$

$u = e^{4x}$

$v = x^3$

$u = e^{4x}$	$v = x^3$
$u^5 = 1024e^{4x}$	$v^{(1)} = 3x^2$
$u^4 = 256e^{4x}$	$v^{(2)} = 6x$
$u^3 = 64e^{4x}$	$v^{(3)} = 6$
$u^2 = 16e^{4x}$	$v^{(4)} = 0$
$u' = 4e^{4x}$	$u^{(5)} = 0$
$u = e^{4x}$	

$y^{(5)} = 1024e^{4x} \cdot x^3 + 1280e^{4x} \cdot 3x^2 + 640e^{4x} \cdot 6x + 160e^{4x} \cdot 6 + 20e^{4x} \cdot 0 + e^{4x} \cdot 0$

$y^{(5)} = x^3 1024e^{4x} + 3x^2 1280e^{4x} + 6x640e^{4x} + 960e^{4x}$

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ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

$$\underbrace{x^2 y''}_{w_1} + \underbrace{xy'}_{w_2} + \underbrace{y'}_{w_3} = 0$$

$$w_1 = x^2 y^{(n+2)} + 2xny^{(n+1)} + 2\frac{n(n-1)}{2!}y^{(n)}$$

$$= x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^{(n)}$$

$$w_2 = xy^{(n+1)} + ny^{(n)}$$

$$w_3 = y^{(n)}$$

$$= x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^{(n)} + xy^{(n+1)} + ny^{(n)} + y^{(n)} = 0$$

$$= x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + [n(n-1) + n + 1]y^{(n)} = 0$$

$$= x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + [n^2 - n + n + 1]y^{(n)} = 0$$

$$= x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + [n^2 + 1]y^{(n)} = 0$$