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Computer Eng
15/ENG02/030
ENG 381

Assigment IV

1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ --- (1)

$(1-x^2)y'' - 2xy' + 2y = 0$ --- (2)

$u = y''$ $\quad V = 1-x^2$

$u^n = y^{(n+2)}$ $\quad V' = 2x$

$V'' = -2$

$V''' = 0$

$y^n = y^{(n+2)}(1-x^2) + n(y^{(n+2-1)} - 2x + n(n-1)y^{(n+2-2)}$

$-2 + n(n-1)(n-2)y^{(n+2-3)} \cdot 0$

$y^n = (1-x^2)y^{(n+2)} - 2xny^{(n+1)} - 2n(n-1)y^{(n)}$ --- (3)

taking the 2nd term $-2x \cdot y'$

$u = y'$ $\quad V = -2x$

$u^n = y^{(n+1)}$ $\quad V' = -2$

$V'' = 0$

$y^n = y^{(n+1)} - 2x + ny^{(n+1-1)} \cdot 2 + n(n-1)y^{(n+1-2)} \cdot 0$

$y^n = -2xy^{(n+1)} - 2ny^{(n)} + 0$ --- (4)

taking the third term $2 \cdot y$

$u = y$ $\quad V = 2$

$u^n = y^n$ $\quad V' = 0$

$y^n = y^n 2 + ny^{(n-1)} 0$

$y^n = 2y^n + ny^{(n-1)} 0$

$y^n = 2y^n$ --- (5)

∴ taking the three equations (3, 4 & 5) we have

$(1-x^2)y^{(n+2)} - 2xny^{(n+1)} - 2n(n-1)y^{(n)} - 2xy^{(n+1)}$

$-2ny^{(n)} + 2y(n) = 0$

$$(1-x^2)y^{(n+2)} - 2x(ny^{(n+1)} + y^{(n+1)}) + y^{(n)}(-2n+2) \quad \text{--- (6)}$$

Now at $x = 0$

$$y^{(n+2)} + 2y^{(n)}(-n+1) = 0$$

$$y^{(n+2)} = -(-n+1)2y^{(n)}$$

$$y^{(n+2)} = (n-1)2y^{(n)} \quad \text{--- (7)}$$

eqn (7) gives the reoccurence equation

at $n = 0$

$$y^2 = (0-1)2y^{(0)}$$

$$(y^2)_0 = -2y_0 \quad \text{--- (7)}$$

at $n = 1$

$$(y^3)_0 = 0 \quad \text{--- (8)}$$

at $n = 2$

$$(y^4)_0 = 2(y^2)_0 = 2(-2y_0) = -4y_0 \quad \text{--- (9)}$$

at $n = 3$

$$(y^5)_0 = 4(y^{(3)})_0 = 4(0) = 0 \quad \text{--- (10)}$$

at $n = 4$

$$(y^6)_0 = 6y^{(4)} = 6(-4y_0) = -24y_0 \quad \text{--- (11)}$$

at $n = 5$

$$(y^7)_0 = 0 \quad \text{--- (12)}$$

at $n = 6$

$$(y^8)_0 = 10(y^6)_0 = 10(-24y_0) = -240y_0 \quad \text{--- (13)}$$

Recall the maclaurins method

$$y(x) = y_0(x) + xy'_0(x) + \frac{x^2}{2!}y''_0(x) + \frac{x^3}{3!}y'''_0(x) + \frac{x^4}{4!}y^4_0(x) + \frac{x^5}{5!}y^5_0(x) + \frac{x^6}{6!}y^6_0(x) \cdots \frac{x^n}{n!}y^n_0(x)$$

Substituting eqn (7, 8, 9, 10, 11, 12, & 13) Into eqn (14) we have

$$y(x) = y_0(x) + xy'_0 + \frac{x^2}{2!}(-2y_0) + \frac{x^3}{3!}(0)$$

$$+\frac{x^4}{4!}(-4y_0)+\frac{x^5}{5!}(0)+\frac{x^6}{6!}(-24y_0)+\frac{x^7}{7!}(0)+\frac{x^8}{8!}(-240y_0)$$

$$y(x)=y_0\left(-\frac{2x^2}{2!}-\frac{x^4}{4!}-\frac{24}{6}x^6-\frac{240}{8!}x^8\right)$$

$$+y_0' x+0$$

$$y(x)=y_0\left(-x^2-\frac{x^4}{6}\right.$$

(2) i)

$3e^{-4t} - 5e^{4t} = f(t)$

$L[ft] = f(s)$

$f(s) = L[3e^{-4t}] - L[5e^{4t}]$

$f[s] = 3\frac{1}{s+4} - 5\frac{1}{s-4}$ //

ii)

$\sin 4t + \cos 4t = f(t)$

$f(s) = L[f(t)]$

$f[s] = L[\sin 4t] + L[\cos 4t]$

$f[s] = \frac{4}{s^2+16} + \frac{s}{s^2+16}$ //

iii) $t^2 + 2t^2 - t + 4 = f(t)$

$L[f(t)] = f(s)$

$f(s) = L[t^2] + 2L[t^2] - L[t] + L[4]$

$f(s) = \frac{2!}{s^3} + 2\left[\frac{2!}{s^3}\right] - \frac{1}{s^2} + \frac{4}{s}$

$f(s) = \frac{2}{s^3} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

iv) $e^{-2t}\cos 5t$

$f(s) = L[e^{-2t}\cos 5t)$

first taking the laplace transform cos 5t

$L[\cos 5t] = \frac{5}{s^2+25}$

$\therefore$

$f(s) = \frac{s+2}{(s+2)^2+25}$ //

v)

$t\sin 2t$

$= \frac{3}{s^2+9}$

recall $= \frac{-d}{ds}\frac{3}{s^2+9}$

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Applying product rule

$$-1\left(\frac{s^2+9(0)-3(2s)}{(s^2+9)^2}\right)$$

$$f(s) = -1\left[\frac{-6s}{(s^2+9)^2}\right]$$

$$f(s) = = \frac{6s}{(s^2+9)^2} //$$

vi)

$$\frac{e^{-t}-e^{-2t}}{t} \Rightarrow \frac{1}{t}\left[e^{-t}-e^{-2t}\right]$$

$$f(t] = \frac{e^{-t}}{t} - \frac{e^{-2t}}{t}$$

$$f(t) = t^{-1}e^{-t} - t^{-1}e^{-2t}$$

$$f(s) = \frac{-1!}{(s+1)^{-1+1}} + \frac{1!}{(s+2)^{-1+2}}$$

$$f(s) = \frac{-1}{1} + \frac{1}{(s+2)}$$

$$f(s) = -1 + \frac{1}{(s+2)}$$

$$f(s) = \frac{1-s-2}{(s+2)} /$$

vii) $e^{4t}\cos 2t$

taking the laplace transform of $\cos 2t$

$$L[\cos 2t) = \frac{s}{s^2+4}$$

$$\therefore L[e^{4t}\cos 2t] = \frac{s-4}{(s-4)^2+4} //$$

viii) $t\sin 2t$

$L[\sin 2t] = \frac{2}{s^2+4}$

$\frac{-d}{ds}\frac{2}{s^2+4}$

$= -\left[\frac{s^2+4(0)-2(2s)}{(s^2+4)^2}\right]$

$= \frac{4s}{(s^2+4)^2}$

ix) t^3+4t^2+5

$f(s) = L[f(t)]$

$f(s) = L[t^3] + L[4t^2] + L[5]$

$f[s] = \frac{3!}{s^4} + \frac{4\,2!}{s^3} + 5/s$

$f[s] = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

x) $t^2\cos t$

$L[\cos t] = \frac{s}{s^2+1}$

$f[s] = \frac{-d}{ds}\left(\frac{-d}{ds}\frac{s}{s^2+1}\right)$

$= -\left[\frac{(s^2+1)(1)-s(2s)}{(s^2+1)^2}\right]$

$= -\left[\frac{s^2+1-4s}{(s^2+1)^2}\right]$

$= \frac{-1-s^2+4s}{(s^2+1)^2} = \frac{4s-s^2-1}{(s^2+1)^2}$

$-\left[\frac{(s^2+1)^2(4-2s)-(4s-s^2-1)(4s^3+4s)}{(s^2+1)^4}\right]$

$= \left[\frac{(s^4+2s^2+1)(4-2s)-(4s-s^2-1)(4s^3+4s)}{(s^2+1)^4}\right]$

$= -\left[\frac{(s^4+2s^2+1)[4-2s]-(16s^3-4s^3-4s^3-4s)}{(s^2+1)^4}\right]$

$= -\left[\frac{4s^4+2s^5-8s^2-12s^3+2s+4}{(s^2+1)^4}\right]$

$= \frac{-4s^4-2s^5+8s^2+12s^3-2s-4}{(s^2+1)^4}$

3 i)

$\frac{s-5}{(s-3)(s-4)}$

$\frac{A}{(s-4)} + \frac{B}{(s-3)} = \frac{s-5}{(s-3)(s-4)}$

$A(s-3)+B(s-4) = s-5$

$A/s = 4 = \frac{4-5}{(4-3)} = \frac{-1}{1} = -1$

$B/s = 3 = \frac{3-5}{(3-4)} = \frac{-2}{-1} = 2$

$A = -1$ & $B = 2$

$f(s) = \frac{-1}{(s-4)} + \frac{2}{(s-3)}$

$f(t) = L^{-1}[f(s)]$

$f(t) = -1e^{4t} + 2e^{3t}$

ii)

$\frac{2s-6}{(s-2)(s-4)}$

$\frac{A}{(s-2)} + \frac{B}{(s-4)} = \frac{2s-6}{(s-2)(s-4)}$

$A/s = 2 = \frac{2(2)-6}{(2-4)} = \frac{-2}{-2} = 1$

$B/s = 4 = \frac{2(4)-6}{(4-2)} = \frac{2}{2} = 1$

$A = 1 \quad \& \quad B = 1$

$$f(s) = \frac{1}{(s-2)} + \frac{1}{(s-4)}$$

$f(t) = L^{-1}[f(s)]$

$f(t) = e^{2t} + e^{4t}$ //

iii)

$$\frac{5s-4}{s(s-4)}$$

$$\frac{A}{s} + \frac{B}{s-4} = \frac{5s-8}{s(s-4)}$$

$$A/s = 0 = \frac{5(0)-8}{(0-4)} = \frac{-8}{-4} = 2$$

$$B/s = 4 = \frac{5(4)-8}{4} = \frac{12}{4} = 3$$

$A = 2 \quad \& \quad B = 3$

$f(s) = 2/s + 3/s-4$

$f(t) = L^{-1}[f(s)]$

$f(t) = 2u(t) = 3e^{4t}$ //

iv)

$$\frac{s^3-3s-4}{(s-3)(s-1)^2}$$

$$\frac{A}{(s-3)} + \frac{B}{(s-1)^2} + \frac{C}{(s-1)} = \frac{s^3-3s-4}{(s-3)(s-1)^2}$$

$$A/s = 3 = \frac{(3)^3-3(3)-4}{(3-1)^2} = \frac{14}{4} = 7/2$$

$$B/s = 1 = \frac{d}{ds} \frac{s^3-3s-4}{(s-3)}$$

$$(s-3)(3s^2-3)-(s^3-3s-4)(1)$$

$$B/s = 1 = \frac{3s^3-3s-9s^2+9-s^3+3s+4}{(s-3)^3}$$

$B/s = 1 = 6/4 = 3/2$

$$C/s = 1 = \frac{s^3-3s-4}{(s-3)} = \frac{(1)^3-3(1)-4}{(1-3)}$$

$$= \frac{1-3-4}{-2}$$

$C/s = 1 = -6/2 = 3$

$$f(s) = \frac{7}{2(s-3)} + \frac{3}{2(s-1)^2} + \frac{3}{(s-1)}$$

$$f(t) = \frac{7}{2}e^{3t} + \frac{3}{2}e^{t} + 3e^{t}$$ //

v)

$$\frac{s-5}{s^2+4s+20}$$

$$\frac{A}{(s+2-j4)} + \frac{B}{(s+2+4j)} = \frac{s-5}{(s+2-j4)(s+2+4j)}$$

$$A/s = -2+4j = \frac{(-2+4j)-5}{(-2+4j+2+4j)} = \frac{7j+4}{8}$$

$B/s = -2-4j = \frac{-2-4j-5}{(-2-4j+2-4j)} = \frac{-7-4j}{-8j} \times \frac{j}{j}$

$B/s = -2-4j = \frac{-7j+4}{8}$

$f(s) = \frac{7j+4}{8(s+2-4j)} - \frac{7i+4}{8(s+2+4j)}$

$f(t) = L^{-1}[f(s)]$

$f(t) = \frac{7j+4}{8} e^{(-2+4j)t} - \frac{7j}{8} + 4 e^{(-2-4j)t}$

$f(t) = \frac{7j+4}{8}\left(e^{(-2+4j)t} - e^{(-2-4j)t}\right)$