Assignment 5

(1) $\frac{dy}{dt} + 3y = e^{-2t}$ given that at $t=0, y=2$

$L\{dy/dt\} = sy_{(s)} - y_{(0)}$

$L\{3y\} = 3y_{(s)}$

$L\{e^{-2t}\} = \frac{1}{s+2}$

$sy_{(s)} - y_{(0)} + 3y_{(s)} = \frac{1}{s+2}$

$sy_{(s)} + 3y_{(s)} - 2 = \frac{1}{s+2}$

$y_{(s)}(s+3) = \frac{1}{s+2} + 2$

$y_{(s)}(s+3) = \frac{1+2(s+2)}{(s+2)}$

$y_{(s)} = \frac{1+2s+4}{(s+2)(s+3)}$

$y_{(s)} = \frac{2s+5}{(s+2)(s+3)}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$

$2s+5 = A(s+3) + B(s+2)$

$2s+5 = As + 3A + Bs + 2B$

$A + B = 2$ — (1)

$3A + 2B = 5$ — (2)

(1) $\times 3$

(2) $\times 1$

$3A + 3B = 6 \quad — ③$

$3A + 2B = 5 \quad — ④$

③ − ④

$B = 1$

from ①

$A + 1 = 2$

$A = 1$

$$\frac{2s+5}{(s+2)(s+3)} = \frac{1}{(s+2)} + \frac{1}{(s+3)}$$

$$L^{-1}\{y_{(s)}\} = L^{-1}\left\{\frac{1}{s+2} + \frac{1}{s+3}\right\}$$

$$y = e^{-2t} + e^{-3t}$$

② $3\frac{dy}{dt} - 6y = \sin 2t$ given that at $t = 0,\ y = 1$

$$L\left\{3\frac{dy}{dt}\right\} = 3\left(sy_{(s)} - y_{(0)}\right)$$

$$L\{-6y\} = -6y_{(s)}$$

$$L\{\sin 2t\} = \frac{2}{s^2+2^2}$$

$$3sy_{(s)} - 3y_{(0)} - 6y_{(s)} = \frac{2}{s^2+2^2}$$

CLT

$$3sy_{(s)} - 6y_{(s)} - 3 = \frac{2}{(s+2)^2}$$

$$y_{(s)}(3s-6) = \frac{2}{(s+2)^2} + 3$$

$$y_{(s)}(3s-6) = \frac{2+3(s+2)^2}{(s+2)^2}$$

$$y_{(s)} = \frac{2+3(s+2)^2}{(s+2)^2(3s-6)}$$

$$\frac{2+3(s+2)^2}{(s+2)^2(3s-6)} = \frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{(3s-6)}$$

$$2+3(s+2)^2 = A(s+2)(3s-6) + B(3s-6) + C(s+2)^2$$

$$2+3s^2+12s+12 = A3s^2 - A12 + 3Bs - 6B + Cs^2 + 4C\cancel{s} + 4C$$

$3A + C = 3$ —— 1

$3B + 4C = 12$ —— 2

$-12A - 6B + 4C = 14$ —— 3

from 1

$3A = 3 - C$

$A = \frac{3-C}{3}$

$3B + 4C = 12$ $\cancel{(4)}$

$-12\left(\frac{3-C}{3}\right) - 6B + 4C = 14$

$-12 + 4C - 6B + 4C = 14$

$-6B + 8C = 28$ —— 4

solving 2 & 4

2 $\times -6$

4 $\times 3$

$-18B - 24C = -72$

$-18B + 24C = 84$

$-48C = -156$

$C = 13/4$

from 2

$3B = 12 - 13$

$B = -\frac{1}{3}$

from ①

$3A = 3 - C$

$3A = 3 - \frac{13}{4}$

$A = -\frac{1}{4/3}$

$A = -\frac{1}{12}$

$$\frac{2+3(s+2)^2}{(s+2)^2(3s-6)} = \frac{-\frac{1}{12}}{(s+2)} - \frac{\frac{1}{3}}{(s+2)^2} + \frac{13/4}{(3s-6)}$$

$$L^{-1}\{y_{(s)}\} = L^{-1}\left\{\frac{-\frac{1}{12}}{(s+2)} - \frac{\frac{1}{3}}{(s+2)^2} + \frac{13/4}{(3s-6)}\right.$$

$$y = -\frac{1}{12}e^{-2t} - \frac{1}{3}te^{-4t} + \frac{13}{12}e^{3t}$$

$$y = -\frac{1}{12}\left(e^{-2t} + 4te^{-4t} - 13e^{3t}\right)$$

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③ $\frac{dy}{dt} - 4y = 8$ given that at $t=0$, $y=2$

$L\{\frac{dy}{dt}\} = sy_{(s)} - y_{(0)}$

$L\{-4y\} = -4y_{(s)}$

$L\{8\} = 8/s$

$sy_{(s)} - y_{(0)} - 4y_{(s)} = 8/s$

$sy_{(s)} - 4y_{(s)} - y_{(0)} = 8/s$

$y_{(s)}(s-4) - 2 = \frac{8}{s}$

$y_{(s)}(s-4) = \frac{8}{s} + 2$

$y_{(s)}(s-4) = \frac{8+2s}{s}$

$y_{(s)} = \frac{8+2s}{s(s-4)}$

$\frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{(s-4)}$

$8+2s = A(s-4) + Bs$

$8+2s = As - 4A + Bs$

$A + B = 2$

$-4A = 8$

$A = -2$

$B = 2+2$

$B = 4$

$\frac{8+2s}{s(s-4)} = \frac{-2}{s} + \frac{4}{(s-4)}$

$$L^{-1}\{y_{(s)}\} = L^{-1}\left\{ \frac{-2}{s} + \frac{4}{(s-4)} \right\}$$

$$y = -2 + 4e^{4t}$$

④ $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ given that at $t=0$, $y=2$, $y'=1$

$$L\left\{\frac{d^2y}{dt^2}\right\} = s^2y_{(s)} - sy_{(0)} - y'_{(0)}$$

$$L\left\{-2\frac{dy}{dt}\right\} = -2sy_{(s)} + 2y_{(0)}$$

$$L\{5y\} = 5y_{(s)}$$

$$L\{e^{2t}\} = \frac{1}{s-2}$$

$$s^2y_{(s)} - sy_{(0)} - y'_{(0)} - 2sy_{(s)} + 2y_{(0)} + 5y_{(s)} = \frac{1}{s-2}$$

$$s^2y_{(s)} - 2sy_{(s)} + 5y_{(s)} - 2s - 1 + 4 = \frac{1}{s-2}$$

$$y_{(s)}(s^2 - 2s + 5) = \frac{1}{s-2} + 2s - 3$$

$$y_{(s)}(s^2-2s+5) = \frac{1 + (2s-3)(s-2)}{(s-2)}$$

$$y_{(s)} = \frac{1 + 2s^2 - 7s + 6}{(s-2)(s^2-2s+5)}$$

$$y_{(s)} = \frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)}$$

$$\frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)} = \frac{A}{(s-2)} + \frac{Bs + C}{(s^2-2s+5)}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$$

$$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$$

$$A + B = 2 \quad \text{—} \ (1)$$

$$-2A - 2B + C = -7 \quad \text{—} \ (2)$$

$$5A - 2C = 7 \quad \text{—} \ (3)$$

from (1)

$$B = 2 - A$$

from (2)

$$-2A - 2(2 - A) + C = -7$$

$$\cancel{-2A} - 4 + \cancel{2A} + C = -7$$

$$C = -3$$

from (3)

$$5A - 2(-3) = 7$$

$$5A = 7 - 6$$

$$A = \frac{1}{5}$$

$$A + B = 2$$

$$\frac{1}{5} + B = 2$$

$$B = 2 - \frac{1}{5}$$

$$B = \frac{9}{5}$$

$$\frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)} = \frac{\frac{1}{5}}{(s-2)} + \frac{\frac{9}{5}s - 3}{(s^2-2s+5)}$$

$$L^{-1}\{y_{(s)}\} = L^{-1}\{\frac{1}{5} \ldots\} + L^{-1}\{\frac{9}{5} \ldots$$

$$\frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{\frac{1}{5}}{(s-2)} + \frac{9/5\, s}{(s^2-2s+5)} - \frac{3}{(s^2-2s+5)}$$

$$= \frac{\frac{1}{5}}{(s-2)} + \frac{9}{5}\left(\frac{s-1+1}{s^2-2s+5}\right) - \frac{3\times\frac{2}{2}}{(s^2-2s+5)}$$

$$= \frac{\frac{1}{5}}{(s-2)} + \frac{9}{5}\left(\frac{s-1+1}{(s-1)^2+4}\right) - \frac{3}{2}\left[\frac{2}{(s-1)^2+4}\right)$$

$$L^{-1}\{y_{(s)}\} = L^{-1}\left\{\frac{\frac{1}{5}}{(s-2)} + \frac{9}{5}\left[\frac{s-1}{(s-1)^2+2^2} + \frac{1\times\frac{2}{2}}{(s-1)^2+2^2}\right] - \frac{3}{2}\left(\frac{2}{(s-1)^2+2^2}\right)\right\}$$

$$\cancel{= \frac{1}{5}e^{2t} + \frac{9}{5}\left[e^t\cos 2t\right.}$$

$$= L^{-1}\left\{\frac{\frac{1}{5}}{(s-2)} + \frac{9}{5}\left[\frac{s-1}{(s-1)^2+2^2} + \frac{1}{2}\left(\frac{2}{(s-1)^2+2^2}\right)\right] - \frac{3}{2}\left(\frac{2}{(s-1)^2+2^2}\right)\right\}$$

$$y = \frac{1}{5}e^{2t} + \frac{9}{5}\left[e^t\cos 2t + \frac{1}{2}\left(e^t\sin 2t\right)\right] - \frac{3}{2}\left(e^t\sin 2t\right)$$

(5) $\frac{d^2y}{dt^2} - \frac{6dy}{dt} + 8y = e^{3t}$ given that at $t=0$, $y=0$, $y'=2$

$$L\left\{\frac{d^2y}{dt^2}\right\} = s^2y_{(s)} - sy_{(0)} - y'_{(0)}$$

$$L\left\{-\frac{6dy}{dt}\right\} = -6sy_{(s)} + 6y_{(0)}$$

$$L\{8y\} = 8y_{(s)}$$

$$L\{e^{3t}\} = \frac{1}{s-3}$$

$$s^2y_{(s)} - sy_{(0)} - y'_{(0)} - 6sy_{(s)} + 6y_{(0)} + 8y_{(s)} = \frac{1}{s-3}$$

$$s^2y_{(s)} - 6sy_{(s)} + 8y_{(s)} - 2 = \frac{1}{s-3}$$

$$y_{(s)}(s^2 - 6s + 8) = \frac{1}{s-3} + 2$$

$$y_{(s)}(s^2 - 6s + 8) = \frac{1 + 2(s-3)}{(s-3)}$$

$$y_{(s)} = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{(s-3)} + \frac{Bs+C}{(s^2-6s+8)}$$

$$2s - 5 = A(s^2-6s+8) + (Bs+C)(s-3)$$

$$2s - 5 = As^2 - 6As + 8A + Bs^2 - 3Bs + Cs - 3C$$

$A + B = 0$ —— (1)

$-6A - 3B + C = 2$ —— (2)

$8A - 3C = -5$ —— (3)

$B = -A$ from (1)

$-6A + 3A + C = 2$

$-3A + C = 2$ —— (4)

$8A - 3C = -5$ —— (5)

(4) $\times -3$

(5) $\times 1$

$9A - 3C = -6$

$8A - 3C = -5$

$A = -1$

$B = 1$

from (4)

$C = 2 - 3$

$C = -1$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{(s-3)} + \frac{s-1}{(s^2-6s+8)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{(s-3)} + \frac{s-1}{(s-2)(s-4)} \quad \text{——} \quad **$$

$$\frac{s-1}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$$

$s - 1 = A(s-4) + B(s-2)$

$s - 1 = As - 4A + Bs - 2B$

$A + B = 1$ —— (1)

$-4A - 2B = -1$ —— (2)

(1) $\times -4$

(2) $\times 1$

$-4A - 4B = -4$

$-4A - 2B = -1$

$-2B = -3$

$B = \frac{3}{2}$ $\quad A = -\frac{1}{2}$

$$\frac{s-1}{(s-2)(s-4)} = \frac{-\frac{1}{2}}{s-2} + \frac{\frac{3}{2}}{s-4}$$

Substitute into **

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{(s-3)} + \left(\frac{-\frac{1}{2}}{(s-2)} + \frac{\frac{3}{2}}{(s-4)} \right)$$

$$L^{-1}\{Y_{(s)}\} = L^{-1}\left\{ \frac{-1}{(s-3)} - \frac{\frac{1}{2}}{(s-2)} + \frac{3/2}{(s-4)} \right\}$$

$$y = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$$

$$y = -\frac{1}{2}\left(2e^{3t} + e^{2t} - 3e^{4t}\right)$$