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1) $dy/dt + 3y = e^{-2t}$ at $t=0, y=2$

$$Sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$$
$$Sy(s) - 2 + 3y(s) = \frac{1}{s+2}$$
$$Sy(s) + 3y(s) = \frac{1}{s+2} + 2$$

$$Sy\ y(s)(s+3) = \frac{1}{s+2} + 2$$
$$y(s) = \frac{1}{(s+2)(s+3)} + \frac{2}{(s+3)}$$

$$\frac{1}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$$
$$1 = A(s+3) + B(s+2)$$

when $s = -3$

$$1 = -B$$
$$B = -1$$

when $s = -2$

$$1 = A$$
$$A = 1$$

$$\frac{1}{(s+2)(s+3)} = \frac{1}{(s+2)} - \frac{1}{(s+3)}$$

$$= L^{-1}\left\{\frac{1}{s+2}\right\} - L^{-1}\left\{\frac{1}{s+3}\right\} + L^{-1}\left\{\frac{2}{s+3}\right\}$$
$$= e^{-2t} - e^{-3t} + 2e^{-3t}$$

2) $3\frac{dy}{dt} - 6y = \sin 2t$, given that at $t=0$, $y=1$

$$3[sy(s) - y(0)] - 6y(s) = \frac{2}{s^2+4}$$

$$3sy(s) - 3 - 6y(s) = \frac{2}{s^2+4}$$

$$3sy(s) - 6y(s) = \frac{2}{6^2+4} + 3$$

$$3y(s)[s-2] = \frac{2}{s^2+4} + 3$$

$$3y(s) = \frac{2}{(s^2+4)(s-2)} + \frac{3}{(s-2)}$$

$$3y(s)[3s-6) = \frac{2}{s^2+4} + 3$$

$$y(s)(3s-6) = \frac{3s^2+14}{s^2+4}$$

$$y(s)(3s-6) = \frac{3s^2+14}{(s^2+4)(3s-6)}$$

$$\frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A}{3s-6} + \frac{B}{s^2+4}$$

$$3s^2+14 = A[s^2+4] + B[3s-6]$$

$$A = 3$$

$$4A - 6B = 14$$

$$-6B = 14-12$$

$$B = -1/2$$

$$y(s) = \frac{3}{3(s-2)} - \frac{1}{3}[s^2+4]$$

$$= e^{2t} - \frac{1}{6}\sin 2t$$

3) $\frac{dy}{dt} - 4y = 8$, given that at $t = 0$, $y = 2$

$$sy(s) - y(0) - 4y(s) = \frac{8}{s}$$

$$sy(s) - 2 - 4y(s) = \frac{8}{s}$$
$$sy(s) - 4y(s) = \frac{8}{s} + 2$$
$$y(s)[s-4) = \frac{8}{s} + 2$$
$$y(s) = \frac{2s+8}{s(s-4)}$$

$$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$2s + 8 = A(s-4) + Bs$$
$$2(0) + 8 = A(0-4)$$
$$A = -2$$
$$2[4] + 8 = A(4-4) + B(4)$$
$$B = 4$$
$$\therefore L^{-1}\left[\frac{-2}{s} + \frac{4}{s-4}\right]$$
$$= -2 + 4e^{4t}$$

4) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ at $t=0, y=2, y'=1$

$s^2y(s) - sy(0) - y'(0) - 2[sy(s) - y(0)] + 5y(s) = \frac{1}{s-2}$

~~$s^2y(s) - sy(0) - 0$~~

$s^2y(s) - 2s - 1 - 2(sy(s)) + 4 + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2s + 1 + 5y(s) = \frac{1}{s-2}$

$s^2y(s) + 5y(s) = 2s - 1 + \frac{1}{s-2}$

$y(s)$

$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2sy(s) + 5y(s) = 2s - 3 + \frac{1}{s-2}$

$y(s)(s^2 - 2s + 5) = \frac{2s(s-2) - 3(s-2) + 1}{s-2}$

$y(s)(s^2 - 2s + 5) = \frac{2s^2 - 4s - 3s + 6 + 1}{s-2}$

$y(s)(s^2 - 2s + 5) = \frac{2s^2 - 7s + 7}{s-2}$

$y(s) = \frac{2s^2 - 7s + 7}{(s^2 - 2s + 5)(s-2)} = \frac{A}{(s-2)} + \frac{B}{(s^2 - 2s + 5)}$

$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + B(s-2)$

$A = 2$

$-2A + B = -7$

$B = -7 + 4 = -3$

$= \frac{2}{s-2} - \frac{3}{s^2 - 2s + 5}$

$= 2e^{2t} - \frac{1}{3}t \sin 2t$

v) $\frac{d^2y}{dt^2} - \frac{6dy}{dt} + 8y = e^{3t}$

$s^2 Y(s) - sy(0) - y'(0) - 6sY(s) + 6Y(0) + 8Y(s) = \frac{1}{s-3}$

$[s^2 - 6s + 8]Y(s) + [6 - s]Y(0) - Y'(0) = \frac{1}{s-3}$

$[s^2 - 6s + 8]Y(s) = \frac{1}{s-3} + 2 = \frac{1 + 2s - 6}{s-3} = \frac{2s-5}{s-3}$

$Y(s) = \frac{2s-5}{[s-3][s-2][s-4]} = \frac{A}{[s-3]} + \frac{B}{[s-2]} + \frac{C}{[s-4]}$

$2s - 5 = A[s-2][s-4] + B[s-3][s-4] + C[s-3][s-2]$

$2s - 5 = A[s^2 - 6s + 8] + B[s^2 - 7s + 12] + C[s^2 - 5s + 6]$

$2(3) - 5 = 1 = A(3-2)(3-4) \Rightarrow A = -1$

$2(4) - 5 = 3 = C(4-3)(4-2) \Rightarrow C = 3/2$

$-6A - 7B - 5C = 2$

$-6(-1) - 7(B) - 5(\frac{3}{2}) = 2$

$-7B = 2 + \frac{15}{2} - 6 = \frac{4 + 15 - 12}{2} = \frac{7}{2} \Rightarrow B = -\frac{1}{2}$

$L^{-1}\left[\frac{-1}{s-3} - \frac{\frac{1}{2}}{(s-2)} + \frac{\frac{3}{2}}{(s-4)}\right] = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$