Name: Patrick Emeka Obianuwezia
Matric NO: 15/SC114/018
Dept: Chemical Engineering

1) $[1-x^2]\frac{d^2y}{dx^2} - 2x\, dy/dx + 2y = 0$

$$\underbrace{[1-x^2]y''}_{w1} - \underbrace{2xy'}_{w2} + \underbrace{2y}_{w3} = 0$$

For w1

$u = y^2 \qquad V = 1-x^2$

$u^n = y^{(n+2)} \qquad V' = -2x$

$u^{(n-1)} = y^{(n+1)} \qquad V^2 = -2$

$u^{(n-2)} = y^{(n)} \qquad V^3 = 0$

$= (1-x^2)(y^{(n+2)}) + n(-2x)(y^{(n+1)}) - \frac{n(n-1)}{\cancel{2}_1}\cancel{2}(y^{(n)})$

$= (1-x^2)(y^{(n+2)}) + -2nx(y^{(n+1)}) - n(n-1)(y^{(n)})$

For w2

$u = y' \qquad V = 2x$

$u^n = y^{n+1} \qquad V' = 2$

$u^{(n-1)} = y^n \qquad V'' = 0$

$2xy^{n+1} + n2y^n$

For w3

$u^n = 2y^n$

LS of SE ... W1 + w2 + w3

$= (1-x^2)(y^{(n+2)}) - \cancel{2nx(y^{(n+1)})} - n(n-1)(y^{(n)}) - \cancel{2xy^{n+1}} - n2y^n + 2y^n$

when $x = 0$

$= y^{(n+2)} - n(n-1)y^{(n)} - n2y^n + 2y^n = 0$

$y^{(n+2)} = n(n-1)\,y^{(n)}[n(n-1) + 2n - 2]$

$y^{(n+2)} = y^{(n)}[n^2 - n + 2n - 2]$

$$y^{(n+2)} = y^{(n)}[n^2+n-2]$$

when $n=0$

$$y^{(2)} = y^{(0)}(-2) = -2y^{(0)}$$

when $n=1$

$$y^{(3)} = y^{(1)}(1^2+1-2) = 0$$

when $n=2$

$$y^{(4)} = y^{(2)}[2^2+2-2) = 4y^{(2)} = -8y^{(0)}$$

when $n=3$

$$y^{(5)} = y^{(3)}[3^2+3-2] = 10y^{(3)} = 0$$

when $n=4$

$$y^{(6)} = y^{(4)}[4^2+4-2] = 18y^{(4)} = -144y^{(0)}$$

when $n=5$

$$y^{(7)} = y^{(5)}[5^2+5-2] = 28y^{(5)} = 0$$

when $n=6$

$$y^{(8)} = y^{(6)}(6^2+6-2) = 40y^{(6)} = -5760y^{(0)}$$

when $n=7$

$$y^{(9)} = y^{(7)}(7^2+7-2) = 54y^{(7)} = 0$$

$$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)}) + \frac{x^4}{4!}(y^{(4)}) + \dots$$

$$y = (y^{(0)})_0 + x(y^{(1)})_0 \cancel{+} - x^2(y^{(0)})_0 + \cancel{x^3}\,0 \cancel{+} - \frac{8x^4}{4!}(y^{(0)})_0$$

$$+ 0 \cancel{+} \cancel{x} - \frac{144x^6}{6!}(y^{(0)})_0 + 0 \cancel{+} - \frac{5760x^8}{8!}(y^{(0)}) + 0$$

$$y = (y^{(0)})_0 + x(y^{(1)})_0 - x^2(y^{(0)})_0 - \frac{x^4}{3}(y^{(0)})_0 - \frac{x^6}{5}(y^{(0)})_0 - \frac{x^8}{7}(y^{(0)})_0$$

2) i) $L\{3e^{-4t} - 5e^{4t}\} = \frac{3}{s+4} - \frac{5}{s-4}$

ii) $L[\sin 4t + \cos 4t] = \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2} = \frac{4+s}{s^2+16}$

iii) $L\{t^3+2t^2-t+4\} = \frac{3!}{s^{3+1}} + \frac{2(2!)}{s^{2+1}} - \frac{1!}{s^{1+1}} + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

iv) $L\{e^{-2t}\cos 5t\} = \frac{s+2}{[s+2]^2+5^2} = \frac{s+2}{s^2+4s+29}$

v) $L\{t\sin 3t\} = -\frac{d}{dx}[f(t)] = -1\cdot\frac{d}{dx}\left[\frac{3}{s^2+3^2}\right]$

$= -\left[\frac{-6s}{(s^2+9)^2}\right] = \frac{6s}{(s^2+9)^2}$

vi) $L\left[\frac{e^{-t}-e^{-2t}}{t}\right] = \frac{\left[\frac{1}{s+1} - \frac{1}{s+2}\right]}{\frac{1}{s^{1+1}}} = \left[\frac{1}{s+1} - \frac{1}{s+2}\right] s^2$

$= \frac{s^2}{(s+1)(s+2)}$

vii) $L\{e^{4t}\cos 2t] = \frac{s}{(s-4)^2+2^2} = \frac{s}{s^2-8s+20}$

viii) $L\{t\sin 2t\} = -1\cdot\frac{d}{dx}\left[\frac{2}{s^2+2^2}\right]$

$= -1\left[\frac{-4s}{(s^2+2^2)^2}\right] = \frac{4s}{[s^2+4]^2}$

ix) $\cancel{t}\ L\{t^3+4t^2+5\} = \frac{3!}{s^{3+1}} + \frac{4(2!)}{s^{2+1}} + \frac{5}{s} = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

x) $L\{e^{3t}(t^2+4)\} = t^2 e^{3t} + 4e^{3t} = \frac{2!}{(s-3)^{1+1}} + \frac{4}{s-3}$

$= \frac{2}{[s-3]^2} + \frac{4}{s-3}$

xi) $t^2 \cos t = [-1]^2 \cdot \frac{d^2}{dx^2}\left[\frac{s}{s^2+1}\right] = \frac{d}{dx}\left[\frac{d}{dx}\left(\frac{s}{s^2+1}\right)\right]$

$= \frac{d}{dx}\left[\frac{s^2+1-2s^2}{(s^2+1)^2}\right] = \frac{d}{dx}\left[\frac{1-s^2}{(s^2+1)^2}\right]$

$= \left[\frac{-2s^5-4s^3-2s-4s+4s^5}{[(s^2+1)^2]^2}\right] = \left[\frac{2s^5-4s^3-6s}{[s^2+1]^4}\right]$

xii) $\frac{\sinh 2t}{t} = \frac{1}{2}\ln(s^2-4)-\ln s$

i) $\dfrac{s-5}{(s-3)(s-4)} = \dfrac{A}{s-3} + \dfrac{B}{s-4}$

$s-5 = A(s-4) + B(s-3)$

$3-5 = A(3-4) \Rightarrow A = 2$

$4-5 = B(4-3) \Rightarrow B = -1$

$L^{-1}\left[\dfrac{2}{s-3} - \dfrac{1}{s-4}\right] = 2e^{3t} - e^{4t}$

ii) $\dfrac{2s-6}{(s-2)(s-4} = \dfrac{A}{s-2} + \dfrac{B}{s-4}$

$2s-6 = A(s-4) + B(s-2)$

$2(2)-6 = A(2-4) \Rightarrow A = 1$

$2(4)-6 = B(4-2) \Rightarrow B = 1$

$L^{-1}\left[\dfrac{1}{s-2} + \dfrac{1}{s-4}\right] = e^{2t} + e^{4t}$

iii) $\dfrac{5s-8}{s(s-4)} = \dfrac{A}{s} + \dfrac{B}{s-4}$

$5s-8 = A(s-4) + Bs$

$5(0)-8 = A(0-4) \Rightarrow A = 2$

$5(4)-8 = B(4) \Rightarrow B = 3$

$L^{-1}\left(\dfrac{2}{s} + \dfrac{3}{s-4}\right) = 2 + 3e^{4t}$

iv) $\dfrac{s^2 3s - 4}{(s-3)(s-1)^2} = \dfrac{A}{s-3} + \dfrac{B}{s-1} + \dfrac{C}{(s-1)^2}$

$s^2 - 3s - 4 = A[s-1]^2 + B[s-3][s-1] + C[s-3]$

$3^2 - 3(3) - 4 = A(3-1)^2 \Rightarrow A = -1$

$1^2 - 3(1) - 4 = C(1-3) \Rightarrow C = 3$

$s^2 - 3s - 4 = [s^2 - 2s + 1]A + [s^2 - 4s + 3]B + [s-3]C$

$-2A - 4B + C = -3$

$-2[-1] - 4[B] + 3 = -3$

$-4B = -3 - 3 - 2 \Rightarrow B = 2$

$\mathcal{L}^{-1}\left[\frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}\right]$

$= -e^{3t} + 2e^{t} + 3te^{t}$

v) $\frac{s-5}{s^2+4s+20} = \frac{s-5}{s^2+4s+4+16} = \frac{s-5}{(s-2)^2+4^2}$

$\frac{s-5}{(s+2)^2+4^2} = [e^{-2t}-7]\cos 4t$