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Computer Engineering
15/ENG02/041

ENG 381 Assignment 4

① $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0$

$w_1$

$(1-x^2)y''$

$u = y'' \qquad v = (1-x^2)$

$u^n = y^{n+2} \qquad v' = -2x$

$v'' = -2$

$v''' = 0$

$u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-1} v''$

$= y^{n+2}(1-x^2) + n(y^{n+1})(-2x) + \frac{n(n-1)}{2!} y^n(-2)$

$= (1-x^2)y^{n+2} - 2xny^{n+1} + (-n^2+n)y^n$

$w_2$

$-2xy'$

$u = y' \qquad v = -2x$

$u = y^{n+1} \qquad v' = -2$

$v'' = 0$

$= u^n v + n u^{n-1} v'$

$= y^{n+1}(-2x) + ny^n(-2)$

$= -2xy^{n+1} - 2ny^n$

$w_3 \quad 2y, \quad u = y, \; u^n = y^n \quad v = 2 \quad v' = 0$

$= 2y^n$

$0 = (1-x^2)y^{n+2} - 2xny^{n+1} + (n-n^2)y^n - 2xy^{n+1} - 2ny^n + 2y^n$

$(1-x^2)y^{n+2} + (y^{n+1})(-2xn - 2x) + y^n(n-n^2-2n+2)$

$(1-x^2)y^{n+2} + y^{n+1}(n+1)(-2x) + y^n(-n^2-n+2) = 0$

$(1-x^2)y^{n+2} - 2xy^{n+1}(n+1) + y^n(-n^2-n+2) = 0$

at $x=0$

$(1-0)y^{n+2} + y^n(-n^2-n+2) = 0$

$(y^{n+2})_0 = -(y^n)_0(-n^2-n+2) = 0$

$(y^{n+2})_0 = (y^n)_0(n^2+n-2)$

at $n=0$

$(y^2)_0 = -(y^0)_0(-0-0+2)$

$= -(y)_0(2)$

$= -2(y)_0$

at $n=1$

$(y^3)_0 = -(y^1)_0(-1-1+2)$

$= +(y^1)_0(-2+2)$

$= 0$

at $n=2$

$(y)^4_0 = (y^2)_0(2^2+2-2)$

$= 4(y^2)_0$

$= 4(-2)(y)_0$

at $n=3$

$(y^5)_0 = (y^3)_0(9+3-2)$

$= (y^3)_0(10)$

$= 0(10) = 0$

at $n=4$

$(y^6)_0 = (y^4)_0(16+4-2)$

$= (y^4)_0(18)$

$= (18)(4)(-2)(y)_0$

at $n=5$

$(y^7)_0 = 0$

$$y^n = (y)_0 + x(y')_0 + \frac{x^2}{2!}(y'')_0 + \frac{x^3}{3!}(y''')_0 + \frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5!}(y^5)_0 + \cdots$$

$$y^n = (y)_0 + x(y')_0 + \left[\frac{x^2}{2}(-2)(y)_0\right] + \left[\frac{x^3}{3!}(0)\right] + \left[\frac{x^4}{\cancel{4}\times 3\times \cancel{2}} \times (\cancel{4})(\cancel{-2})(y)_0\right] + \left[\frac{x^5}{5!}(0)\right] + \left[\frac{x^6}{\cancel{6}\times 5\times \cancel{4}\times \cancel{3}\times \cancel{2}}(\cancel{18})(\cancel{4})(\cancel{-2})(y)_0\right]$$

$$y^n = (y)_0 + x(y')_0 + \left[-2x^2(y)_0\right] + 0 + \left[\frac{-x^4}{3}(y)_0\right] + 0 + \left[\frac{-x^6}{5}(y)_0\right]$$

$$y^n = (y)_0 + x(y')_0 - 2x^2(y)_0 - \frac{x^4}{3}(y)_0 - \frac{x^6}{5}(y_0)$$

$$y^n = (y)_0\left[1 - 2x^2 - \frac{x^4}{3} - \frac{x^6}{5} + \cdots\right] + x(y')_0$$

(2) (1) $3e^{-4t} - 5e^{4t}$

$$= \frac{3}{s+4} - \frac{5}{s-4}$$

$$\frac{3(s-4) - 5(s+4)}{(s+4)(s-4)}$$

$$= \frac{3s - 12 - 5s - 20}{s^2 - 16}$$

$$= \frac{-2s - 22}{s^2 - 16}$$

(ii) $\sin 4t + \cos 4t$

$$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$$

$$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$$

$$= \frac{4+s}{s^2+16}$$

(iii) $t^3 + 2t^2 - t + 4$

$$t^n = \frac{n!}{s^{n+1}}$$

$$= \frac{3!}{s^4} + \frac{2!(2)}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$= \frac{6+4s-s^2+4s^3}{s^4}$$

(iv) $e^{-2t}\cos 5t$

$$= \frac{s+2}{(s+2)^2+25}$$

(v) $t\sin 3t$

$$= (-1)^1 \frac{dF(s)}{ds^2}$$

$$F(s) = \frac{3}{s^2+3^2} = \frac{3}{s^2+9}$$

$\frac{dF(s)}{ds^2} =$ using quotient rule

$u = 3$ $\quad$ $v = s^2+9$

$du = 0$ $\quad$ $dv = 2s$

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$$= \frac{V \, dy/dx - U \, dy/dx}{V^2}$$

$$= \frac{s^2+9(0) - 3(2s)}{(s^2+9)^2}$$

$$= \frac{-3(2s)}{(s^2+9)^2}$$

$$\frac{dF(s)}{ds} = \frac{-6s}{(s^2+9)^2}$$

$$\therefore (-1)\frac{dF(s)}{ds} = -\left(\frac{-6s}{(s^2+9)^2}\right) = \frac{6s}{(s^2+9)^2}$$

(i) $$\frac{e^{-t} - e^{-2t}}{t}$$

$$= \frac{e^{-t}}{t} - \frac{e^{-2t}}{t}$$

$$L\left(\frac{e^{-t}}{t}\right)$$

$\Rightarrow$ taking the limit at $t = 0$

$$= \frac{e^{-0}}{0} = \text{indeterminate}$$

using l'hospital rule

$$= \frac{-e^{-t}}{1} = \frac{-e^{0}}{1} = \frac{0}{1} = 0$$

$$\therefore \int_{s=0}^{\infty} F(s)$$

$$F(s) = \frac{1}{s+1}$$

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$$= \int_{s=\sigma}^{\infty} \frac{1}{\sigma+1} \, d\sigma$$

$$= \ln(\sigma+1) \Big|_{s=\sigma}^{\infty}$$

$$= \ln(\infty+1) - \ln(s+1)$$

$$\ln \infty - \ln(s+1)$$

$$= \ln \frac{\infty}{s+1}$$

$$\frac{e^{-2t}}{t}$$

$$F(s) = \frac{1}{s+2}$$

$$= \int_{\sigma=s}^{\infty} \frac{1}{\sigma+2} \, d\sigma$$

$$= \ln(\sigma+2) \Big|_{\sigma=s}^{\infty}$$

$$= \ln(\infty+2) - \ln(s+2)$$

$$= \ln \infty - \ln(s+2)$$

$$= \ln\left(\frac{\infty}{s+2}\right)$$

$$\therefore L\left[\frac{e^{-t}}{t} - \frac{e^{-2t}}{t}\right] = \ln\left(\frac{\infty}{s+1}\right) - \ln\left(\frac{\infty}{s+2}\right)$$

$$= \ln\left(\frac{\infty}{s+1} \div \frac{\infty}{s+2}\right)$$

$$= \ln\left(\frac{\cancel{\infty}}{s+1} \times \frac{s+2}{\cancel{\infty}}\right)$$

$$= \ln\left(\frac{s+2}{s+1}\right)$$

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vii) $e^{4t} \cos 2t$

$$= \cancel{\frac{(s+4)}{(s+4)^2}} = \frac{(s-4)}{(s-4)^2+2^2}$$

$$= \frac{s-4}{s^2-8s+16+4} = \frac{s-4}{s^2-8s+20}$$

viii) $t \sin 2t$

$$= (-1) \frac{d\,F(s)}{ds^n}$$

$$F(s) = \frac{2}{s^2+2^2} = \frac{2}{s+4}$$

$$\frac{d\,F(s)}{ds} = \text{using quotient rule}$$

$u = 2 \qquad v = s^2+4$

$du = 0 \qquad dv = 2s$

$$= v\,du/dx - u\,dv/dx$$

$$= \frac{(s^2+4)(0) - 2(2s)}{(s^2+4)^2}$$

$$= \frac{-4s}{(s^2+4)^2}$$

$$\therefore (-1) \frac{d\,F(s)}{ds^n} = -\left(\frac{-4s}{(s^2+4)^2}\right)$$

$$= \frac{4s}{(s^2+4)^2}$$

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m) $t^3 + 4t^2 + 5$

$$= \frac{3!}{s^4} + \frac{4 \times 2!}{s^3} + \frac{5}{s}$$

$$= \cancel{3\times} \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

$$= \frac{6 + 8s + 5s^3}{s^4}$$

$$= \frac{1}{s^4}(5s^3 + 8s + 6)$$

② $e^{3t}(t^2 + 4)$

$$= t^2 e^{3t} + 4e^{3t}$$

$$= \frac{2}{(s-3)^3} + \frac{4}{s-3}$$

$$= \frac{2 + 4(s-3)^2}{(s-3)^3}$$

(su) $t^2 \cos t$

$$= (-1)^2 \frac{d^2 F(s)}{ds^2}$$

$$F(s) = \frac{s}{s^2+1^2} = \frac{s}{s^2+1}$$

$$\frac{dF(s)}{ds} =$$

let $u = s$ $\quad v = s^2+1$

$du = 1$ $\quad dv = 2s$

$$= \frac{s^2+1(1) - s(2s)}{(s^2+1)^2} = \frac{s^2+1-2s^2}{(s^2+1)^2}$$

$$\cancel{= \frac{2s}{(s^2+1)^2}} = \frac{-s^2+1}{(s^2+1)^2}$$

$\frac{d^2F(s)}{ds^2} =$

let $u = -s^2+1$ $\quad v = (s^2+1)^2$

$dv =$

$u = s^2+1 \quad du = 2s$

$v = u^2 \quad dv = 2u$

$dv = \frac{du}{ds} \times \frac{dv}{du}$

$= 2s \cdot (2u)$

$= 4su$

$du = -2s \quad dv = 4s(s^2+1)$

$\therefore \frac{(+s^2+1)^2(-2s) - (-s^2+1)(s^2+1)4s}{(s^2+1)^4}$

$= \frac{\cancel{[s^2+1]}[s^2+1(-2s) - (-s^2+1)4s]}{(s^2+1)^{\cancel{4}\,3}}$

$= \frac{(-2s)(s^2+1) - 4s(-s^2+1)}{(s^2+1)^3}$

$= \frac{-2s^3 - 2s + 4s^3 - 4s}{(s^2+1)^3}$

$= \frac{2s^3 - 6s}{(s^2+1)^3}$

$= \frac{2s(s^2-6)}{(s^2+1)^3}$

$\therefore L[t^2\cos t] = \frac{2s(s^2-6)}{(s^2+1)^3}$

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x11) $\frac{\sinh 2t}{t}$

using l'hospital rule

$\frac{2\cosh 2t}{1}$

taking lim at $t \to 0$

$\frac{2\cosh 0}{1} = 2$

$= \int_{\sigma=s}^{\infty} F(s)$

$F(s) = \frac{2}{s^2 - 2^2} = \frac{2}{s^2-4}$

$= \int_{s=\sigma}^{\infty} \frac{2}{\sigma^2 - 4}$

$= 2\int_{s=\sigma}^{\infty} \frac{1}{\sigma^2 - 2^2} \, d\sigma$

$= \cancel{2}\left(\frac{1}{\cancel{2}(2)}\right) \ln\left[\frac{\sigma - 2}{\sigma + 2}\right]$

$= \frac{1}{2} \ln\left[\frac{\sigma - 2}{\sigma + 2}\right]\Big|_{\sigma=s}^{\infty}$

$= \frac{1}{2}\ln\left(\frac{\infty - 2}{\infty + 2}\right) - \frac{1}{2}\ln\left[\frac{s-2}{s+2}\right]$

$= \frac{1}{2}\ln\left(\frac{\infty}{\infty}\right) - \frac{1}{2}\ln\left[\frac{s-2}{s+2}\right]$

$= \ln 1 - \frac{1}{2}\ln\left[\frac{s-2}{s+2}\right]$

$= \ln\left[\frac{s+2}{s-2}\right]^{-1/2}$

(3) Convert the following functions to time (t) domains:

(i) $\frac{s-5}{(s-3)(s-4)}$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$$

$s-5 = A(s-4) + B(s-3)$

at $s = 4$

$4-5 = A(4-4) + B(4-3)$

$-1 = B(1)$

$B = -1$

at $s=3$

$3-5 = A(3-4) + B(3-3)$

$-2 = A(-1)$

$A = 2$

$= \frac{2}{s-3} - \frac{1}{s-4}$

$= 2e^{3t} - e^{-4t}$

(ii) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$2s-6 = A(s-4) + B(s-2)$

$2s-6 = As - 4A + Bs - 2B$

$A + B = 2$ --- (i)

$-4A - 2B = -6$ --- (ii)

$A = 2 - B$ --- (iii)

$-4(2-B) - 2B = -6$

$-8 + 4B - 2B = -6$

$2B = -6 + 8$

$B = 1$

$A = 2 - B$

$= 2 - 1 = 1$

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$$= \frac{1}{s-2} + \frac{1}{s-4}$$

$$= e^{2t} + e^{4t}$$

(iii) $$\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$5s-8 = A(s-4) + Bs$$

at $s=0$

$$-8 = A(-4)$$

$$A = 2$$

at $s=4$

$$20-8 = 4B$$

$$12 = 4B$$

$$B = 3$$

$$= \frac{2}{s} + \frac{3}{s-4}$$

$$= 2 + 3e^{4t}$$

(iv) $$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{(s-3)} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$$

$$s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$$

at $s=1$

$$1-3-4 = C(1-3)$$

$$-6 = C(-2)$$

$$C = 3$$

at $s=3$

$$3^2-3(3)-(4) = A(3-1)^2$$

$$9-9-4 = A(2)^2$$

$$-4 = 4A$$

$$A = -1$$

$$s^2-3s-4 = A(s^2-2s+1) + B(s^2-4s+3) + C(s-3)$$

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$s^2-3s-4 = As^2 - 2As + A + Bs^2 - 4Bs + 3B + Cs - 3C$

$A + B = 1 \quad --- (i)$

$-2A - 4B + C = -3$

$A + 3B - 3C = -4$

$B = 1 - A \quad --- (ii)$

$= 1 - (-1)$

$= 1 + 1$

$= 2$

$= \frac{-1}{s-3} + \frac{2}{s+1} + \frac{3}{(s+1)^2}$

$= -e^{3t} + 2e^{-t} + 3t\,e^{-t}$

(2) $\frac{s-5}{s^2+4s+20}$

$= \frac{s-5}{s^2+4s+4+16} = \frac{s-5}{s^2+2s+2s+4+16}$

$= \frac{s-5}{(s+2)^2+16}$

$= \frac{s-5}{(s+2)^2+4^2}$

$$= \frac{s - 5 + 2 - 2}{(s+2)^2 + 4^2}$$

$$= \frac{s+2}{(s+2)^2+4^2} + \left(\frac{-5-2}{(s+2)^2+4^2}\right)$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{(s+2)^2+4^2}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \left[\frac{\cancel{7}}{(s+2)^2+4^2} \times \frac{4}{\cancel{7}} \times \frac{7}{4}\right]$$

$$= \frac{s+2}{(s+2)^2+4^2} - \left[\frac{4}{(s+2)^2+4^2} \times \frac{7}{4}\right]$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4}\left(\frac{4}{(s+2)^2+4^2}\right)$$

$$= e^{-2t}\cos 4t - \frac{7}{4} e^{-2t} \sin 4t$$