NAME: ALAOMA .G. CHISOM
MATRIC NO: 15/SCI01/007
DEPT: COMPUTER ENGINEERING
COURES: ENG 381

ASSIGNMENT 4

ALAOMA · G. CHISOM
15/SCI01/007
COMP ENG

Ass(4)

(3)

(i) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$\frac{s-5}{(s-3)(s-4)} = \frac{A(s-4) + B(s-3)}{(s-3)(s-4)}$

$s - 5 = A(s-4) + B(s-3)$

$s = 4,$

$4 - 5 = A(4-4) + B(4-3)$

$B = -1$

$s = 3,$

$3 - 5 = A(3-4)$

$A = 2$

$= \frac{2}{s-3} - \frac{1}{s-4} = 2\left(\frac{1}{s-3}\right) - \frac{1}{s-4}$

$= 2e^{3t} - e^{4t}$

(ii) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$\frac{2s-6}{(s-2)(s-4)} = \frac{A(s-4) + B(s-2)}{(s-2)(s-4)}$

$2s - 6 = A(s-4) + B(s-2)$

$s = 2,$

$A = 1$

$s = 4$

$2 = 2B$

$B = 1$

$= \frac{1}{s-2} + \frac{1}{s-4} = e^{2t} + e^{4t}.$

(iii) $\frac{5s-8}{s(s-4)}$ =

$= \frac{A}{s} + \frac{B}{s-4}$

$\frac{5s-8}{s(s-4)} = \frac{A(s-4)+Bs}{s(s-4)}$

$5s-8 = A(s-4)+Bs$

$s=0$

$\frac{-8}{-4} = \frac{-4A}{-4}$

$A = 2$

$s=4$

$\frac{12}{4} = B$

$B = 3$

$= \frac{2}{s} + \frac{3}{s-4}$

$= 2 + 3e^{4t}$.

(iv) $\frac{(s^2-3s-4)}{(s-3)(s-1)^2}$

$= \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$

$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A(s-1)^2 + B(s-3)(s-1) + C(s-3)}{(s-3)(s-1)^2}$

$s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$

$s=3$

$3^2-3(3)-4 = A(3-1)^2$

$-4 = 4A$

$A = -1$

$s=1$

$1^2-3(1)-4 = A(1-1)^2 + B(s-3)(s-1) + C(1-3)$

$-6 = -2C$

$C = 3$

$s = 0$

$0^2 - 3(0) - 4 = A(0-1)^2 + B(0-3)(0-1) + C(0-3)$

$-4 = A + 3B - 3C$

$-4 = -1 + 3B - 3(3)$

$-4 = -1 + 3B - 9$

$-4 = -10 + 3B$

$-4 + 10 = 3B$

$\frac{6}{3} = \frac{3B}{3}$ $\quad B = 2$

$= -\frac{1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}$

$= -e^{3t} + 2e^t + 3te^t$

(v) $\frac{s-5}{s^2+4s+20}$

$\frac{s-5}{s^2+4s+20} = \frac{As+B}{s^2+4s+20}$

$s - 5 = As + B$

$A = 1$

$B = -5$

$\frac{s-5}{s^2+4s+20} = \frac{s-5}{(s+4)^2+4^2}$

$= e^{5t}\cos 4t$

1. $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{6} u^{n-3} v^3 + \ldots$

$(1-x^2)y'' - 2xy' + 2y = 0$

for sub (1)

$(1-x^2)y''$

$v = 1-x^2 \qquad u^n = y^{n+2}$

$v' = -2x$

$v'' = -2$

$= (1-x^2)y^{n+2} - n y^{n+2-1}(-2x) + \frac{n(n-1)}{2} y^{n+2-2}(-2) + 0$

$= (1-x^2)y^{n+2} + 2xn y^{n+1} - n(n-1)y^n$

for sub (2)

$-2xy$

$v = -2x \qquad u^n = y^{n+1}$

$v' = -2$

$v'' = 0$

$= -2x y^{n+1} + n y^{n+1-1}(-2) + 0$

$= -2x y^{n+1} - 2n y^n$

for sub (3)

$2y$

$v = 2 \qquad y^n = u^n$

$v' = 0$

$= 2y^n$

$y^n = (1-x^2)y^{n+2} + 2xn y^{n+1} - n(n-1)y^n - 2x y^{n+1} - 2n y^n + 2y^n$

at $x = 0$

$= y^{n+2} - n(n-1)y^n - 2n y^n + 2y^n$

$= y^{n+2} + 2y^n - (n^2+n)y^n - 2n y^n$

$y^{n+2} = (n^2+n)y^n + 2n y^n - 2y^n$

$y^{n+2} = y^n(n^2+3n-2)$

$(y^{n+2})_0 = (y^n)_0 (n^2 + 3n - 2)$

$n = 0$

$(y^2)_0 = -2(y)_0$

$n = 1$

$(y^3)_0 = 3(y')_0$

$n = 2$

$(y^4)_0 = 8(y^2)_0 = -16(y)_0$

$n = 3$

$(y^5)_0 = 48(y')_0$

$n = 4$

$(y^6)_0 = 416(y)_0$

$n = 5$

$(y^7)_0 = 1824(y')_0$

$$y = y_0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3) + \frac{x^4}{4!}(y^4) + \frac{x^5}{5!}(y^5) +$$

$$y = y_0 + x(y') - \frac{x^2}{2} \cdot -2(y) + \frac{x^3}{6} \cdot 3(y') + \frac{x^4}{24} \cdot -16(y) + \frac{x^5}{120} \cdot 1824(y')$$

$$y = (y_0)_0 \left\{ 1 + x^2 - \frac{2}{3}x^4 \right\} + (y')_0 \left\{ x + \frac{x^3}{2} + \frac{76}{5}x^5 + \dots \right\}$$

(2)

(a) $3e^{-4t} - 5e^{4t}$

$$= 3 \cdot \frac{1}{s+4} - 5 \cdot \frac{1}{s-4}$$

$$= \frac{3}{s+4} - \frac{5}{s-4}$$

(b) $\sin 4t + \cos 4t$

$$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2} = \frac{4}{s^2+16} + \frac{s}{s^2+16}$$

(c) $t^3 + 2t^2 - t + 4$

$$\frac{3!}{s^4} + 2\left(\frac{2!}{s^3}\right) - \frac{1}{s^2} + \frac{4}{s}$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

(d) $e^{-2t}\cos 5t$

$$= \frac{(s+a)}{(s+a)^2 + a^2} = \frac{s+2}{(s+2)^2 + 5^2} = \frac{s+2}{(s+2)^2 + 25}$$

(e) $t \sin 3t$

$$= -\frac{d}{ds}\left(\frac{a}{s^2 + a^2}\right) = -\frac{d}{ds}\left(\frac{3}{s^2+9}\right)$$

$$\frac{d}{ds}\left(3 \cdot (s^2+9)^{-1}\right) = 3\frac{d}{ds}(s^2+9)^{-1}$$

$$\frac{d}{ds} = \frac{(s^2+9)0 - 3(3s)}{(s^2+9)^2} = \frac{-9s}{(s^2+9)^2}$$

(f) $\dfrac{e^{-t} - e^{-2t}}{t}$ *

$$L(e^{-t}) = \frac{1}{s+1} \quad ; \quad L(-e^{-2t}) = \frac{1}{s+2}$$

$$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = (-1)^1 \frac{d}{ds}\left[\frac{1}{s^2+3s+2}\right]$$

$$= \frac{2s+3}{(s^2+3s+2)^2}$$

(g) $e^{4t}\cos 2t$

$$= \frac{s-4}{(s-4)^2 + 4}$$

(h) $t \sin 2t$

$$L(t \sin 2t) = \frac{-\partial}{\partial s}\left(\frac{2}{s^2+4}\right) = \frac{2\partial}{\partial s}(s^2+4)^{-1}$$

$$\frac{\partial}{\partial s}\left(\frac{2}{s^2+4}\right) = \frac{(s^2+4)0 - 2(2s)}{(s^2+4)^2}$$

$$= \frac{-4s}{(s^2+4)^2}$$

(i) $t^3 + 4t^2 + 5$

$$= \frac{3!}{s^4} + 4\left(\frac{2!}{s^3}\right) + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

(j) $(e^{st}, t^2) + 4$

$$= \frac{2!}{(s+9)^3} + \frac{4}{s}$$

$$= \frac{2}{(s-3)^3} + \frac{4}{s}$$

(K) $t^2 \cos t$

$$L(t^2 \cos t) = (-1)^2 \frac{\partial}{\partial s}\left(\frac{s}{s^2+1}\right)$$

$$= \frac{(s^2+)1 - s(2s)}{(s^2+1)^2} = \frac{1-s^2}{(s^2+1)^2}$$

(L) $\frac{\sin h 2t}{t}$

$$= -1\left[\frac{4s}{(s^2-4)^2}\right] = \frac{4s}{(s^2-4)^2}$$