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## ASSIGNMENT 5

$$(1) \frac{dy}{dt} + 3y = e^{-3t}$$

$$t=0, y=2$$

$$s(y)s - y(0) + 3y(s) = \frac{1}{s+2}$$

$$s(y)s - 2 + 3y(s) = \frac{1}{s+2}$$

$$y(s)(s+3) = \frac{2}{1} + \frac{1}{s+2}$$

$$y(s)(s+3) = \frac{2s+5}{s+2}$$

$$y(s) = \frac{2s+5}{(s+3)(s+2)} = \frac{A}{s+3} + \frac{B}{s+2}$$

$$2s+5 = A(s+2) + B(s+3)$$

$$s = -2$$

$$2(-2)+5 = B(-2+3)$$

$$1 = B$$

$$\therefore B = 1$$

$$s = -3$$

$$2(-3)+5 = A(-3+2)$$

$$-1 = -A$$

$$A = 1$$

$$= \frac{1}{s+3} + \frac{1}{s+2}$$

$$= e^{-3t} + e^{-2t}$$

$$(3) \frac{dy}{dt} - 4y = 8$$

$$t=0, y=2$$

$$s y(s) - y(0) - 4y(s) = \frac{8}{s}$$

$$s y(s) - 2 - 4y(s) = \frac{8}{s}$$

$$y(s)(s-4) = \frac{8}{s} + \frac{2}{1} = \frac{2s+8}{s}$$

$$y(s) = \frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$= \frac{A(s-4) + Bs}{s(s-4)}$$

$$= 2s + 8 = A(s-4) + Bs$$

$$s=4$$

$$2(4) + 8 = 4B$$

$$\frac{16}{4} = B$$

$$B = 4$$

$$s=0$$

$$2(0) + 8 = A(0-4) + B(0)$$

$$\frac{8}{-4} = \frac{-4A}{-4}$$

$$A = -2$$

$$= \frac{-2}{s} + \frac{4}{s-4}$$

$$= \mathcal{L}^{-1}\left(\frac{-2}{s}\right) + \mathcal{L}^{-1}\left(\frac{4}{s-4}\right)$$

$$= -2 + 4e^{4t}$$

$$= 4e^{4t} - 2$$

$$5) \quad \frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$$

$$t=0, y=0, y'=2$$

$$s^2y(s) - sy(0) - y'(0) - 6(sy(s) - y(0)) + 8y(s) = \frac{1}{s-3}$$

$$s^2y(s) - 0 - 2 - 6sy(s) + 12 + 8y(s) = \frac{1}{s-3}$$

$$s^2y(s) - 6sy(s) + 8y(s) + 10 = \frac{1}{s-3}$$

$$y(s) (s^2 - 6s + 8) = \frac{10}{1} + \frac{1}{s-3} = \frac{-10s + 31}{s-3}$$

$$y(s) = \frac{-10s + 31}{(s^2 - 6s + 8)(s-3)} = \frac{-10s + 31}{(s-2)(s-4)(s-3)}$$

$$y(s) = \frac{-10s + 31}{(s^2 - 6s + 8)(s-3)} = \frac{A}{s-2} + \frac{B}{s-4} + \frac{C}{s-3}$$

$$\Rightarrow -10s + 31 = A(s-4)(s-3) + B(s-2)(s-3) + C(s-2)(s-4)$$

$$s = 4$$

$$-10(4) + 31 = B(4-2)(4-3)$$

$$B = \frac{-9}{2}$$

$$s = 3$$

$$-10(3) + 31 = C(3-2)(3-4)$$

$$1 = -C$$

$$C = -1$$

$$s = 2$$

$$-10(2) + 31 = A(2-4)(2-3)$$

$$A = \frac{11}{2}$$

$$= \left( \frac{11}{2(s-2)} - \frac{9}{2(s-4)} \right) - \frac{1}{s-3}$$

$$= \frac{1}{2} \left( \frac{11}{s-2} - \frac{9}{s-4} \right) - \frac{1}{s-3}$$

$$= \frac{1}{2} (11e^{2t} - 9e^{4t}) - e^{3t}$$

$$= 2^{-1} (11e^{2t} - 9e^{4t}) - e^{3t}$$

$$4) \frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$$

$$t=0, y=2, y'=1$$

$$s^2 y(s) - sy(0) - y'(0) - 2(sy(s) - y(0)) + 5y(s) = \frac{1}{s-2}$$

$$s^2 y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$$

$$s^2 y(s) - 2sy(s) + 5y(s) - 2s + 3 = \frac{1}{s-2}$$

$$y(s) \cdot \{s^2 - 2s + 5\} = 2s - 3 + \frac{1}{s-2}$$

$$y(s) \{s^2 - 2s + 5\} = \frac{2s^2 - 7s + 7}{s-2}$$

$$y(s) = \frac{2s^2 - 7s + 7}{(s^2 - 2s + 5)(s-2)} = \frac{A}{s-2} + \frac{B}{s-2} + \frac{C}{s-2}$$

$$2) \quad 3 \frac{dy}{dt} - 6y = \sin 2t$$

$$3 (sY(s) - y(0)) - 6Y(s) = \frac{2}{s^2 + 4}$$

$$3 (sY(s) - 1) - 6Y(s) = \frac{2}{s^2 + 4}$$

$$3 (sY(s)) - 6Y(s) = \frac{1}{1} + \frac{2}{s^2 + 4}$$

$$3 \cdot Y(s) (s - 6) = \frac{(s+2)^2 + 2}{(s+2)^2}$$

$$Y(s) = \frac{(s^2 + 4) + 2}{(s-6)(s+2)^2} = \frac{A}{s-6} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$Y(s) = \frac{(s^2 + 4) + 2}{(s-6)(s+2)^2} = \frac{A(s+2)^2 + B(s-6)(s+2) + C(s-6)}{(s-6)(s+2)^2}$$

$$(s^2 + 4) + 2 = A(s+2)^2 + B(s-6)(s+2) + C(s-6)$$

$s = -2$	$s = 6$	$s = 0$
$10 = \frac{-8C}{-8}$	$42 = \frac{64A}{64}$	$6 = 4A - 12B - 6C$
$C = \frac{-5}{4}$	$A = \frac{21}{32}$	$6 = 4\left(\frac{21}{32}\right) - 12B - 6\left(\frac{-5}{4}\right)$
		$6 = \frac{84}{32} + \frac{30}{4} - 12B$
		$6 = \frac{81}{8} - 12B$

$$6 - \frac{81}{8} = -12B$$

$$-\frac{33}{8} = B \quad \therefore B = \frac{11}{32}$$

$$= \frac{21}{32} \left( \frac{1}{s-6} \right) + \frac{11}{32} \left( \frac{1}{s+2} \right) + \frac{5}{4} \left( \frac{1}{s^2 + 4} \right)$$

$$= \frac{21}{32} e^{6t} + \frac{11}{32} e^{-2t} - \frac{5}{4} \sin 2t$$