IZOKPU IROHAN
15/ENG03/068
CIVIL ENGINEERING

1. $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$\cancel{W = t}$

$(1-x^2)y'' - 2xy' + 2y = 0$

$W = (1-x^2)y''$

$V = (1-x^2)$, $V' = -2x$, $V'' = -2$, $V''' = 0$

$u = y''$, $u^n = y^{(n+2)}$

$W^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 + \ldots$

$= y^{n+2}(1-x^2) + n y^{n+2-1} \cdot -2x + \frac{n(n-1)}{\cancel{2}} y^{n+2-2} \cdot \cancel{-2} + \frac{n(n-1)(n-2)}{6} y^{n+2-3} \times 0 + \ldots$

$= y^{n+2}(1-x^2) + 2nx y^{n+1} + n(n+1) y^n + 0$

$W = 2xy$

$V = 2x$, $V' = 2$, $V'' = 0$

$u = y'$, $u^n = y^{n+1}$

$W^n = u^n v + n u^{(n+1)} y' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \ldots$

$= y^n \cdot 2x + n y^{n-1} \cdot 2 + \frac{n(n-1)}{2} y^{n-2} \cdot 0$

$= y^{n+1} 2x + 2n y^{n}$

$w^1 = 2y$

$v = 2, \; v^1 = 0$

$u = y, \; u^n = y^n$

$$w = u^n v + n u^{n-1} v^1 + \frac{n(n-1)}{2} u^{n-2} v^2 + \dots$$

$$= y^n \cdot 2 + n y^{n-1} \cdot 0 +$$

$$= 2y^n$$

$$y^{n+2}(1-x^2) + 2nxy^{n+1} \cancel{+} n(n\cancel{+}1) y^n - (y^{n+1} 2x + 2ny^{n-1}) + 2y^n = 0$$

at $x = 0$

$$y^n = y^{n+2}(1-0^2) + 2n(0)y^{n+1} \cancel{+} n(n\cancel{+}1)y^n - (y^{n+1} 2(0) + 2ny^{n-1}) + 2y^n = 0$$

$$= y^{n+2} + 0 \cancel{+} (n^2 \cancel{+} n) y^n - (0 + 2ny^{n-1}) + 2y^n -$$

$$= y^{n+2} \cancel{+} (n^2 \cancel{+} n) y^n - 2ny^{n-1} + 2y^n$$

$$\cancel{= y^{n+2} + (n^2 + n + 2) y^n - 2ny^{n-1}}$$

$$= y^{n+2} + (n^2 + n + 2) y^n - 2ny^{n-1} = 0$$

$$y^{n+2} = (n^2 \cancel{+} n - 2) y^n \cancel{+} 2ny^{n-1}$$

$$y^{n+2} = (n^2 - n + 2n - 2) y^n$$

$$(y^{n+2})_0 = [(n^2 + n - 2) y^n]_0$$

when $n = 1, 2, 3, 4 \dots$ etc, when $n = 0$

$y^2 = -2y_0$

when $n = 1$,

$(y^3)_0 = = 0y^1_0$

when $n=2$,

$$(y^4)_0 = (4y^2)_0 = 4(-2y)_0 = -8y_0$$

when $n=3$

$$(y^5)_0 = (10y^3)_0 = 10(0) = 0y'_0$$

when $n=4$

$$(y^6)_0 = (18y^4)_0 = 18(4)(-2)y_0 = -144y_0$$

when $n=5$

$$(y^7)_0 = 28y^5 = 28(10)(0)y_0 = 0y'_0$$

Maclaurin's series

$$y = (y_0) + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5!}(y^5)_0 + \cdots$$

$$= y_0 + x(y')_0 + \frac{x^2}{2!}(-2y_0)_0 + \frac{x^3}{3!}(0)_0 + \frac{x^4}{4!}(-8y_0) + \frac{x^5}{5!}(0)_0 + \cdots$$

$$= y_0 + xy'_0 + \frac{x^2}{\cancel{2}}(-\cancel{2}y)_0 + 0 + \frac{x^4}{24}(-8y_0) + 0 + \cdots$$

$$= y_0 + xy'_0 - 2x^2y_0 - \frac{3x^4}{3}y_0$$

$$= y_0(1 - 2x^2 - \frac{3x^4}{3}) + xy'_0$$

NIGERIAN UNIVERSITIES ENGINEERING STUDENT ASSOCIATION

AFE BABALOLA UNIVERSITY, ADO EKITI

2017/2018 NUESA EXECUTIVE COUNCIL

21) $L\{3e^{-4t} - 5e^{4t}\}$

$L\{3e^{-4t}\} = 3L\{e^{-4t}\} = 3 \times \frac{1}{s-(-4)} = \frac{3}{s+4}$

$L\{-5e^{4t}\} = -5L\{e^{4t}\} = -5 \times \frac{1}{s-4} = \frac{-5}{s-4}$

$= \frac{3}{s+4} - \frac{5}{s-4}$

$= \frac{3(s-4)-5(s+4)}{(s+4)(s-4)} = \frac{3s-12-5s-20}{(s+4)(s-4)} = \frac{-2s-32}{(s+4)(s-4)}$

$= \frac{-2s-32}{s^2-4s+4s-16} = \frac{-2s-32}{s^2-16}$

22) $L\{\sin 4t + \cos 4t\}$

$L\{\sin 4t\} = \frac{4}{s^2+4^2} = \frac{4}{s^2+16}$

$L\{\cos 4t\} = \frac{s}{s^2+4^2} = \frac{s}{s^2+16}$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$

$= \frac{4+s}{s^2+16}$

3 $\mathcal{L}\{t^3 + 2t^2 - t + 4\}$

$\mathcal{L}\{t^3\} + \mathcal{L}\{2t^2\} + \mathcal{L}\{-t\} + \mathcal{L}\{4\}$

$\mathcal{L}\{t^3\} + 2\mathcal{L}\{t^2\} - 1\mathcal{L}\{t\} + \mathcal{L}\{4\}$

$$\frac{3!}{s^{3+1}} + 2\left[\frac{2!}{s^{2+1}}\right] - 1\left[\frac{1!}{s^{1+1}}\right] + \frac{4}{s}$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$= \frac{1}{s}\left[\frac{6}{s^3} + \frac{4}{s^2} - \frac{1}{s} + 4\right]$$

4. $\mathcal{L}\{e^{-2t}\cos 5t\}$

$$= \frac{s-(-2)}{(s-(-2))^2 + 5^2} = \frac{s+2}{(s+2)^2 + 25} = \frac{s+2}{s^2 + 4s + 4 + 25} = \frac{s+2}{s^2 + 4 + 29}$$

5 $\mathcal{L}\{t^n \sin 3t\} = (-1)^n \frac{d^n \sin 3t}{ds^n}$

$$= (-1)^1 \frac{d}{ds}\left[\frac{3}{s^2 + 3^2}\right]$$

$$= (-1) \frac{d}{ds}\left[\frac{3}{s^2 + 9}\right]$$

$$= -1 \left[\frac{(s^2+9)0 - 3(2s)}{(s^2+9)^2}\right]$$

$$= -1 \left[\frac{-6s}{(s^2+9)^2}\right.$$

$$= \frac{6s}{(s^2+9)^2}$$

6. $e^{-t} \cdot \cancel{\frac{-e^{-2t}}{t}}$

7. $\cancel{e^{4t}\cos 2t}$ $L\{e^{4t}\cos 2t\}$

$$= \frac{s-4}{(s-4)^2+2^2} = \frac{s-4}{s^2-8s+16+4} = \frac{s-4}{s^2-4s+20}$$

8. $L\{t\sin 2t\} = (-1)^1 \frac{d}{ds}\left[\frac{2}{s^2+4}\right]$

$$= -1 \cdot \left[\frac{(s^2+4)0 - 2(2s)}{(s^2+4)^2}\right]$$

$$= -1\left[\frac{-4s}{(s^2+4)^2}\right]$$

$$= \frac{4s}{(s^2+4)^2}$$

9. $\cancel{t^3+4t^2+5}$ $L\{t^3+4t+5\}$

$L\{t^3\} + 4L\{t\} + L\{5\}$

$$\frac{3!}{s^{3+1}} + 4\left(\frac{1!}{s^{1+1}}\right) + \frac{5}{s}$$

$$\frac{6}{s^4} + \frac{4}{s^2} + \frac{5}{s}$$

1a $L\{e^{3t}(t^2+4)\}$

$L\{t^2e^{3t}+4e^{3t}\}$

$L\{t^2e^{3t}\} + L\{4e^{3t}\}$

$(-1)^2\frac{d}{ds}(e^{3t}]$

$= \cancel{-(-te^{3t}}$

$=(-1)^2\frac{d}{ds}(e^{3t}]$

$= 1 \cdot \left(\frac{1}{s-3}\right]$

$= \frac{(s-3)0 - 1(1)}{(s-3)^2}$

$= \frac{0-1}{(s-3)^2} = \frac{-1}{(s-3)^2}$

b $e^{-t}-e^{-2t}$

$L\{e^{-t}-e^{-2t}\}$

$= \frac{1}{s+1} - \frac{1}{s+2}$

$L\left\{\frac{e^{-t}-e^{-2t}}{t}\right\} = \int_{\sigma=s}^{\infty} \frac{1}{s+1} - \frac{1}{s+2}$

$\int_{\sigma}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+2}$

$\ln(\sigma+1) - \ln(\sigma+2)$

$\ln\left[\frac{\sigma+1}{\sigma+2}\right] = \ln\left[\frac{s+1}{s+2}\right]$

$= -\ln\left[\frac{s+1}{s+2}\right] = \ln\left[\frac{s+1}{s+2}\right]^{-1}$

$= \ln\left[\frac{s+2}{s+1}\right]$ //

1i) $\cancel{\frac{\sinh 2t}{t}}$ $L\left\{\frac{\sinh 2t}{t}\right\}$

$L\{\sinh 2t\} = \frac{2}{s^2-4}$

$L\left\{\frac{\sinh 2t}{t}\right\} = \int_{s \to s}^{\infty} \frac{2}{s^2-4} = 2\int_{s}^{\infty} \frac{1}{s^2-4}$

$= 2\left[\frac{1}{\cancel{2}} \tan^{-1}\frac{s}{\cancel{2}}\right]_s^{\infty} = \left[\frac{\tan^{-1} s}{2}\right]_s^{\infty}$

$= \tan^{-1}\frac{\infty}{2} - \tan^{-1}\frac{s}{2}$

$= -\tan^{-1}\frac{s}{2} = \left[\tan^{-1}\frac{s}{2}\right]^{-1}$

$= \tan^{-1}\frac{2}{s}$

3i) $L^{-1}\left\{\frac{s-5}{(s-3)(s-4)}\right\}$

$\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$s-5 = A(s-4) + B(s-3)$

at $s=4$

$4-5 = A(4-4) + B(4-3)$

$-1 = A(0) + B$

$B = -1$

NIGERIAN UNIVERSITIES ENGINEERING STUDENT ASSOCIATION
AFE BABALOLA UNVERSITY, ADO EKITI.

2017/2018
NUESA EXECUTIVE COUNCIL

at $s = 3$

$3-5 = A(3-4) + B(3-3)$

$-2 = A(-1) + B(0)$

$A = 2$

$$L^{-1}\left[\frac{A}{s-3} + \frac{B}{s-4}\right] = L^{-1}\left[\frac{2}{s-3} - \frac{1}{s-4}\right]$$

$$= 2e^{3t} - e^{4t}$$

2) $\frac{2s-6}{(s-2)(s-4)}$

$$L^{-1}\left[\frac{A}{s-2} + \frac{B}{s-4}\right]$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$2s-6 = A(s-4) + B(s-2)$

at $s = 4$

$2(4) - 6 = A(4-4) + B(4-2)$

$2 = 2B$

$B = 1$

at $s = 2$

$2(2) - 6 = A(2-4) + B(2-2)$

$-2 = -2A$

$A = 1$

$$x(t) = L^{-1}\left\{\frac{1}{s-2} + \frac{1}{s-4}\right\}$$

$$x(t) = e^{2t} + e^{4t}$$

iii) $\frac{5s-8}{s(s-4)}$

$$L^{-1}\left\{\frac{A}{s} + \frac{B}{s-4}\right\}$$

$$\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$5s - 8 = A(s-4) + B(s)$

at $s = 0$

$5(0) - 8 = A(0-4) + B(0)$

$-8 = -4A$

$A = \frac{-8}{-4}$

$= 2$

$L^{-1}\left\{\frac{2}{s} + \frac{3}{s-4}\right\}$

$x(t) = 2 + 3e^{4t}$

(iv) $L\left\{\frac{s^2-3s-4}{(s-3)(s-1)^2}\right\}$

$x(t) = L^{-1}\left[\frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}\right]$

$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$

$s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1)^2 + C(s-3)$

$s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$

at $s = 3$

$(3)^2 - 3(3) - 4 = A(3-1)^2 + B(3-3)(3-1) + C(3-3)$

$-4 = 4A$

$A = \frac{-4}{4} = -1$

at $s = 1$

$(1)^2 - 3(1) - 4 = A(1-1)^2 + B(1-3)(1-1) + C(1-3)$

$-6 = -2C$

$C = 3$

$s^2 - 3s - 4 = As^2 - 2As + A + Cs - 3 - Bs^2 - 4Bs - 3B$

$A + 3B - 3C = -4$

$-1 + 3B - 3(3) = -4$

$3B = 10 - 4$

$3B = 6$

$B = 2$

$\therefore x(t) = L^{-1}\left\{\frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}\right\}$