AGBEDE GOODNESS TERIMAUN
15/ENG03/003
CIVIL ENGINEERING
ENG 381 (ENGINEERING MATHEMATICS)

1. $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)\frac{d^2y}{dx^2} = (1-x^2)y''$

W_1 $U = y^2$ $U^n = y^{(n+2)}$
$V = 1-x^2$ $V' = -2x$ $V'' = -2$ $V^3 = 0$

$W^n = U^n V + nU^{(n-1)}V' + \frac{n(n-1)U^{(n-2)}}{2!}V^2 + \frac{n(n-1)(n-2)U^{(n-3)}}{3!}V^3$

$= y^{(n+2)}.(1-x^2) + ny^{(n+1)}. -2x + \frac{n(n-1)}{\cancel{2}}y^n.\cancel{2} + 0$

$W^n = (1-x^2)y^{(n+2)} - 2xny^{(n+1)} - n(n-1)y^n$
$= (1-x^2)y^{(n+2)} - 2xny^{(n+1)} - (n^2-n)y^n$

$W_2 = -2x\frac{dy}{dx} = -2xy'$

$U = y'$ $U^n = y^{n+1}$
$V = -2x$ $V' = -2$ $V^2 = 0$
$= y^{(n+1)}. -2x + ny^n. -2 + 0$
$= -2xy^{(n+1)} - 2ny^n$

$W_3 = 2y$
$U = y$ $U^n = y^n$
$V = 2$ $V' = 0$
$y^n.2 + 0$
$= 2y^n$

$= (1-x^2)y^{(n+2)} - 2xny^{(n+1)} - 2xy^{(n+1)} - (n^2-n)y^n - 2ny^n + 2y^n$
$= (1-x^2)y^{(n+2)} - 2xy^{(n+1)}(n+1) - y^n(n^2-n+2n-2)$
$= (1-x^2)y^{(n+2)} - (n+1)2xy^{(n+1)} - y^n(n^2+n-2)y^2$

at $x = 0$
$= (1-0^2)y^{(n+2)} - (n+1)2(0)y^{(n+1)} - (n^2+n-2)y^n$
$y^{(n+2)} - (n^2+n-2)y^n$
$[y^{(n+2)}]_0 = (n^2+n-2)y^n$

at $n = 0$
$[y^2]_0 = -2[y^0]_0$
at $n = 1$
$[y^3]_0 = 0$
at $n = 2$

$[y^4]_0 = 4[y^2]_0 = 4\times -2[y^0]_0 = -8[y^0]_0$

at $n=3$

$[y^5]_0 = 10[y^3]_0 = 10\times 0 = 0$

at $n=4$

$[y^6]_0 = 18[y^4]_0 = 18\times -8[y^0]_0 = -144[y^0]_0$

$y = y_0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}[y^3]_0 + \cdots \frac{x^r}{r!}(y^r)_0$

$= y_0 + x(y')_0 + \frac{x^2}{2!}\cdot -2[y^0]_0 + \cancel{\frac{x^3}{3!}\cdot 0} + \frac{x^4}{4!}\cdot -8[y^0]_0 +$

$\frac{x^5}{5!}\cdot 0 + \frac{x^6}{6!}\cdot -144[y^0]_0.$

$=[y^0]_0 + x(y')_0 - x^2[y^0]_0 - \frac{x^4[y^0]_0}{3} - \frac{x^6}{5}(y^0]_0$

$y_0[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}] + x[y']_0$

2. $L[3e^{-4t} - 5e^{4t}]$.

$L[3e^{-4t}] - L[5e^{4t}]$

$\frac{3}{s+4} - \frac{5}{s-4}$

ii $L[\sin 4t + \cos 4t]$

$L[\sin 4t] + L[\cos 4t]$

$\frac{4}{s^2+16} + \frac{4}{s^2+16}$

$= \frac{4+s}{s^2+16}$

iii $L[t^3 + 2t^2 - t + 4]$

$t^n = \frac{n!}{s^{n+1}}$

$= \frac{3!}{s^4} + 2\left[\frac{2!}{s^3}\right] - \frac{1}{s^2} + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

iv $L[e^{-2t}\cos 5t]$

$L[\cos 5t] = \frac{s}{s^2+25}$

$L[e^{-2t}\cos 5t] = \frac{s+2}{(s+2)^2+25}$

$= \frac{s+2}{(s+2)^2+25}$

(v) $t\sin 3t$

$\because L[\sin 3t] = \frac{3}{s^2+9}$

$L[t\sin 3t] = -1\frac{d}{ds}\left[\frac{3}{s^2+9}\right]$

$u = 3 \quad du = 0$

$v = s^2+9 \quad dv = 2s$

$-1\left[\frac{0-6s}{(s^2+9)^2}\right] = \frac{6s}{(s^2+9)^2}$

(vi) $e^{-t} - e^{-2t}$

$L[e^{-t} - e^{-2t}]$

$= \frac{1}{s+1} - \frac{1}{s+2}$

$L\left[\frac{e^{-t}-e^{-2t}}{t}\right] = \int_{\sigma=s}^{\infty} \frac{1}{s+1} - \frac{1}{s+2}$

$= \int_{s}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+2}$

$\ln(\sigma+1) - \ln(\sigma+2)$

$= \ln\left[\frac{\sigma+1}{\sigma+2}\right]$

$= \ln\left[\frac{s+1}{s+2}\right]$

$= -\ln\left[\frac{s+1}{s+2}\right]$

$= \ln\left[\frac{s+1}{s+2}\right]^{-1} = \ln\left[\frac{s+2}{s+1}\right]$

(vii) $L[e^{4t}\cos 2t]$

$L[\cos 2t] = \frac{s}{s^2+4}$

$L[e^{4t}\cos 2t] = \frac{s-4}{(s-4)^2+4}$

(viii) $t\sin 2t$

$L[\sin 2t] = \frac{2}{s^2+4}$

$L[t\sin 2t] = -1\frac{d}{ds}\left[\frac{2}{s^2+4}\right]$

$u = 2 \quad du = 0$

$v = s^2+4 \quad dv = 2s$

$-1\left[\frac{0-4s}{(s^2+4)^2}\right] = \frac{4s}{(s^2+4)^2}$

(ix) t^3+4t^2+5

$L[t^3] + L[4t^2] + L[5]$

$= \frac{3!}{s^4} + 4\left[\frac{2!}{s^3}\right] + \left[\frac{5}{s}\right]$

$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

x) $t^2\cos t$

$L[\cos t] = \frac{s}{s^2+1}$

$L[t^2\cos t] = (-1)^2\frac{d}{ds}\left[\frac{s}{s^2+1}\right]$

$\frac{d}{ds}\left[\frac{s}{s^2+1}\right]$

$u = s \quad du = 1$

$v = s^2+1 \quad dv = 2s$

$-1\left[\frac{0-2s}{(s^2+1)}\right] = \frac{2s}{s^2+4}$

xi. $e^{3t}(t^2+4)$

$L[t^2e^{3t}+4e^{3t}]$

$L[t^2e^{3t}] = \frac{2}{s^3}$

$= \frac{2}{(s-3)^3}$

$L[4e^{3t}]$

$\frac{4}{s-3}$

$e^{3t}(t^2+4) = \frac{2}{(s-3)^3} + \frac{4}{s-3}$

(xii) $\frac{\sinh 2t}{t}$

$L[\sinh 2t] = \frac{2}{s^2-4}$

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$$L\left[\frac{\sinh 2t}{t}\right] = \int_{s=s}^{\infty} \frac{2}{s^2-4} = 2\int_{s}^{\infty} \frac{1}{s^2-4}$$

$$= 2\left[\frac{1}{2}\frac{\tan^{-1}s}{2}\right]_s^{\infty} = \left[\frac{\tan^{-1}s}{2}\right]_s^{\infty}$$

$$\frac{\tan^{-1}\infty}{2} - \tan^{-1}\frac{s}{2}$$

$$-\tan^{-1}\frac{s}{2}$$

$$\left[\tan^{-1}\frac{s}{2}\right]^{-1}$$

$$= \tan^{-1}\frac{2}{s}$$

(1.3) Convert the following to time (t) domain:

(i) $\frac{s-5}{(s-3)(s-4)}$

$$L^{-1}\left[\frac{A}{s-3} + \frac{B}{s-4}\right]$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$$

$$s-5 = A(s-4) + B(s-3)$$

at $s=4$.

$$4-5 = A(4-4) + B(4-3)$$

$$-1 = A(0) + B(1)$$

$$-1 = 0 + B$$

$$B = -1$$

at $s=3$.

$$3-5 = A(3-4) + B(3-3)$$

$$-2 = -A + B(0).$$

$$A = 2.$$

$$L^{-1}\left[\frac{2}{s-3} + \frac{-1}{s-4}\right]$$

$$x(t) = 2e^{3t} - e^{4t}$$

(ii) $\frac{2s-6}{(s-2)(s-4)}$

$$L^{-1}\left[\frac{A}{(s-2)} + \frac{B}{(s-4)}\right]$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$$

$$2s-6 = A(s-4) + B(s-2)$$

at $s=4$

$$2(4)-6 = A(4-4) + B(4-2)$$

$$2 = 2B$$

$$B = 1.$$

at $s=2$

$$2(2)-6 = A(2-4) + B(2-2)$$

$$-2 = -2A$$

$$A = 1$$

$$x(t) = L^{-1}\left[\frac{1}{(s-2)} + \frac{1}{(s-4)}\right]$$

$$x(t) = e^{2t} + e^{4t}.$$

(iii) $\frac{5s-8}{s(s-4)}$

$$L^{-1} = \left[\frac{A}{s} + \frac{B}{s-4}\right]$$

$$\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$5s-8 = A(s-4) + Bs$$

$$A + B = 4.$$

$$5(4)-8 = A(4-4) + B(4)$$

$$12 = 4B$$

$$B = \frac{12}{4} = 3$$

at $s=0$

$$5(0)-8 = A(0-4) + B(0)$$

$$-8 = -4A$$

$$A = \frac{-8}{4} = -2$$

$$L^{-1}\left[\frac{2}{s} + \frac{3}{s-4}\right]$$

$$L^{-1}\left[\frac{2}{s} + \frac{3}{s-4}\right].$$

$$x(t) = 2 + 3e^{4t}$$

v) $\dfrac{S^2-3S-4}{(S-3)(S-1)^2}$

$x(t) = L^{-1}\left[\dfrac{A}{S-3} + \dfrac{B}{S-1} + \dfrac{C}{(S-1)^2}\right]$

$S^2-3S-4 = A(S-1)^2 + B(S-3)(S-1) + C(S-3)$

$S^2-3S-4 = A(S-1)^2 + B(S-3)(S-1) + C(S-3)$

at $S=3$

$(3)^2 - 3(3) - 4 = A(3-1)^2 + B(3-3) + C(3-3)$

$-4 = 4A$

$A = \dfrac{-4}{4} = -1$

At $S=1$

$1^2 - 3(1) - 4 = A(1-1)^2 + B(1-3)(1-1) + C(1-3)$

$-6 = -2C$

$C = \dfrac{6}{2}$

$C = 3$

$S^2-3S-4 = AS^2 - 2AS + A + CS - C3 + BS^2$
$- B4S + B3$

$A + 3B - 3C = -4$

$-1 + 3B - 3(3) = -4$

$3B = 10-4$

$3B = 6$

$B = \dfrac{6}{3}$

$B = 2$