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15/ENG03/022
Civil Engineering (1)

ENG 381

$(1-x^2)\frac{\partial^2 y}{\partial x^2} - 2x\frac{\partial y}{\partial x} + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0$

$(1-x^2)y''$

$v = 1-x^2, \quad v' = -2x, \quad v'' = -2, \quad v''' = 0$

$u = y'' \quad u^n = y^{n+2}$

$u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} u^{n-3} v'''$

$= y^{n+2}\cdot(1-x^2) + n y^{n+2-1}\cdot -2x + \frac{n(n-1)}{\cancel{2}} y^{n+2-2}\cdot -\cancel{2}$

$= y^{n+2}\cdot(1-x^2) - 2xny^{n+1} - n(n-1)y^n$

$-2xy'$

$v = -2x, \quad v' = -2 \quad v'' = 0$

$u = y' \quad u^n = y^{n+1}$

$= y^{n+1}\cdot -2x + n y^{n+1-1}\cdot -2$

$= -2xy^{n+1} - 2ny^n$

$2y$

$v = 2 \quad v' = 0$

$u = y \quad u^n = y^n$

$= 2y^n$

$= y^{n+2}(1-x^2) - 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n$

at $x = 0$

$y^{n+2}(1-0^2) - 2(0)ny^{n+1} - n(n-1)y^n - 2(0)y^{n+1} - 2ny^n + 2y^n$

$= y^{n+2} - n(n-1)y^n - 2ny^n + 2y^n$

$= y^{n+2} + (-n(n-1) - 2n + 2)y^n$

$y^{n+2} - (n^2 - n + 2n - 2)y^n = 0$

$y^{n+2} - (n^2 + n - 2)y^n = 0$

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Maclaurins principle

$= y_0 + x y'_0 + \frac{x^2}{2!}[y''_0] + \frac{x^3}{3!}[y'''_0] \dots$

at $n = 1, 2, 3, 4, 5$

$y^{n+2} = (n^2+n-2)y^n$

$n = 0$

$y^2 = -2y^0$

$n = 1$

$y^3 = (1^2+1-2)y^1 = 0$

$n = 2$

$y^4 = (2^2+2-2)y''$

$= 4y'' = 4(-2)y^0 = -8y^0$

$n = 3$

$y^5 = (3^2+3-2)y'''$

$= 10y''' = 10\times 0 = 0$

$n = 4$

$y^6 = [4^2+4-2)y^4$

$= 18y^4 = 18\times 4y'' = 72(-2y^0) = -144y^0$

$n = 5$

$y^7 = [5^2+5-2)y^5$

$= 28y^5 = 28\times 0 = 0$

$= y_0 + xy'_0 + \frac{x^2}{\cancel{2}!}[-\cancel{2}y^0] + \cancel{\frac{x^3}{3!}}[0] + \frac{x^4}{4!}[-8y^0] + \cancel{\frac{x^5}{5!}[0]}$

$= y_0 + xy'_0 - x^2y^0 - \frac{x^4}{3}y_0$

$= \left\{1 - x^2 - \frac{x^4}{3}\right\}y_0 + xy'_0$

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a. $3e^{-4t} - 5e^{4t}$

$L\{3e^{-4t}\} - L[5e^{4t}]$

$= 3\left[\frac{1}{s+4}\right] - 5\left[\frac{1}{s-4}\right]$

$= \frac{3}{s+4} - \frac{5}{s-4}$

$\frac{3(s-4)-5(s+4)}{(s+4)(s-4)} = \frac{3s-12-5s-20}{(s+4)(s-4)} = \frac{-2s-32}{(s+4)(s-4)}$

ii) $\sin 4t + \cos 4t$

$\frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

$\frac{4+s}{s^2+4^2} = \frac{4+s}{s^2+4^2}$

iii) $t^3 + 2t^2 - t + 4$

$L[t^3] = \frac{3!}{s^4} = \frac{6}{s^4}$

$2L[t^2] = \frac{2\cdot 2!}{s^3} = \frac{2}{s^3}$

$L[t] = \frac{1!}{s^2} = \frac{1}{s^2}$

$L[4] = \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

$\frac{6+4s-s^2+4s^3}{s^4}$

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b) $e^{-2t}\cos 5t$

$$= \frac{s+2}{(s+2)^2+5^2} = \frac{s+2}{(s+2)^2+25} = \frac{s+2}{s^2+4s+4+25} = \frac{s+2}{s^2+4s+29}$$

c) $t\sin 3t$

$$= (-1)^1 \frac{\partial}{\partial s}\left[\frac{3}{s^2+3^2}\right]$$

$$= -\left[\frac{6s}{(s^2+9)^2}\right] = \frac{-6s}{(s^2+9)^2}$$

d) $\frac{e^{-t}-e^{-2t}}{t}$

$$\frac{e^{-(0)}-e^{-2(0)}}{0} = \frac{1-1}{0} = \frac{0}{0}.$$

$$= \cancel{e^{-t}} - \cancel{e^{-2t}}/t^{-1}$$

$$L[\cancel{e^{-t}}] = \frac{1}{s+1}$$

using L'hopital

$$= \frac{-e^{-t}+2e^{-2t}}{1} = \frac{-e^{-(0)}+2e^{-2(0)}}{1}$$

$$L[e^{-2t}t^{-1}] = (-1)^{-1}\frac{\partial}{\partial s}$$

$$= \frac{-1+2}{1}$$

$$L[e^{-t} - e^{-2t}]$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$= \int_{s=\sigma}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+2} = \cancel{\int_s^{\infty}\left[\frac{1}{\sigma+1} - \int\frac{1}{\sigma+2}\right]}$$

$$= \int_s^{\infty}[\ln\sigma+1 - \ln\sigma+2]$$

$$= \cancel{\int \ln(\sigma+1)} \quad \cancel{\ln\sigma+1} \quad \ln\left(\frac{\sigma+1}{\sigma+2}\right)_s^{\infty}$$

$$\ln\frac{\infty+1}{\infty+2} - \ln\left(\frac{s+1}{s+2}\right)$$

$$-\ln\left(\frac{s+1}{s+2}\right)$$

$$\ln\left(\frac{s+1}{s+2}\right)^{-1}$$

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vii) $e^{4t} \cos 2t$

$$= \frac{s-4}{(s-4)^2 + 2^2} = \frac{s-4}{(s-4)^2 + 4} = \frac{s-4}{s^2 - 8s + 16 + 4} = \frac{s-4}{s^2 - 8s + 20}$$

viii) $t \sin 2t$

$$(-1)^2 \frac{\partial}{\partial s}\left[\frac{2}{s^2+4}\right]$$

$$= \left[\frac{4s}{(s^2+4)^2}\right] = \frac{4s}{s^4 + 8s^2 + 16}$$

ix) $= t^3 + 4t^2 + 5$

$$= L\left[t^3\right] + L\left[4t^2\right] + L\left[5\right]$$

$$= \frac{3!}{s^4} + 4\left[\frac{2!}{s^3}\right] + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s} = \frac{6 + 8s + 5s^3}{s^4}$$

x) $e^{3t}(t^2 + 4)$

$$= t^2 e^{3t} + 4e^{3t}$$

$$L\left[t^2 e^{3t}\right]$$

$$= (-1)^2 \frac{\partial^2}{\partial s^2}\left[\frac{1}{s-3}\right]$$

$$= \frac{\partial}{\partial s}\left[\frac{1}{(s-3)^2}\right] = \frac{\partial}{\partial s}\left[\frac{1}{s^2 - 6s + 9}\right]$$

xi) $t^2 \cos t$

$$(-1)^2 \frac{\partial}{\partial s^2}\left[\frac{s}{s^2+1^2}\right]$$

$$-1^2 \frac{\partial}{\partial s^2}\left[\frac{s}{s^2+1}\right] = \frac{\partial}{\partial s}\left[s^2 - 2s^2 + 1\right]$$

$\frac{d}{dx}\tan^{-1} = \frac{1}{1+x^2}$

② $\frac{2}{4+s^2}$

$2\tan^{-1}$

X(ii) $\frac{\sinh 2t}{t}$

limit $t \to 0$

$= \frac{\sinh 2(0)}{0} = 0$

using l'hopital rule

$= \frac{2\cosh 2t}{1} = 2\cosh 2(0) = 2$

$\int_{\sigma=2}^{\infty} \frac{2}{s^2-4}\, d\sigma$

$\int_{\sigma=2}^{\infty} \frac{2}{-4+s^2}\, d\sigma$

$-2\int_{\sigma=3}^{\infty} \frac{1}{4+s^2}\, d\sigma$

$\frac{-2}{4}\left[\tan^{-1}\sigma\right]_{\sigma=5}^{\infty}$

$-\frac{1}{2}\tan^{-1}\infty -$

$\frac{1}{2}\tan^{-1}s$

$\left(\tan^{-1}\frac{s}{2}\right)^{-1}$

$= \tan^{-1}\frac{2}{s}$

③

3) i) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$\frac{s-5}{\cancel{(s-3)(s-4)}} = \frac{A(s-4)+B(s-3)}{\cancel{(s-3)(s-4)}}$

$s-5 = A(s-4)+B(s-3)$

let $s = 4$

$4-5 = A(4-4)+B(4-3)$

$B = -1$

let $s = 3$

$3-5 = A(3-4)+B(3-3)$

$-2 = -A$

$A = 2$

$\frac{2}{s-3} - \frac{1}{s-4}$

$= 2e^{3t} - e^{4t}$

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(3)

i) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$\frac{2s-6}{\cancel{(s-2)(s-4)}} = \frac{A(s-4)+B(s-2)}{\cancel{(s-2)(s-4)}}$

$2s-6 = A(s-4)+B(s-2)$

$s=4$

$2(4)-6 = A(4-4)+B(4-2)$

$2=2B$

$B=1$

$s=2$

$2(2)-6 = A(2-4)+B(2-2)$

$-2=-2A$

$A=1$

$= \frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$

$L^{-1}\left[\frac{1}{s-2} + \frac{1}{s-4}\right]$

$= e^{2t} + e^{4t}$

ii) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$\frac{5s-8}{\cancel{s(s-4)}} = \frac{A(s-4)+Bs}{\cancel{s(s-4)}}$

$5s-8 = A(s-4)+B(s)$

at $s=4$

$5(4)-8 = A(4-4)+4B$

$20-8 = 4B$

$B = 12/4 = 3$

at $s=0$

$5(0)-8 = A(0-4)+B(0)$

$-8 = -4A$

$A=2$

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(2) -

$= \frac{2}{s} + \frac{3}{s-4}$

$L^{-1}\left[\frac{2}{s} + \frac{3}{s-4}\right]$

$= 2 + 3e^{4t}$

10) $\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$

$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A(s-1)^2 + B(s-3)(s-1) + C(s-3)}{(s-3)\cancel{(s-1)^2}}$

$s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$

at $s=1$

$1^2-3(1)-4 = A(1-1)^2 + B(1-3)(1-1) + C(1-3)$

$-6 = -2C$

$C = 3$

at $s=3$

$3^2-3(3)-4 = A(3-1)^2 + B(3-3)(3-1) + C(3-3)$

$-4 = 4A$

$A = -1$

~~s^2-3s~~ at $s=0$

$0^2-3(0)-4 = A(0-1)^2 + B(0-3)(0-1) + C(0-3)$

$-4 = A + 3B - 3C$

$-4 = -1 + 3B - 3\times 3$

$-4 = -1 + 3B - 9$

$-4 = -10 + 3B$

$-4 + 10 = 3B$

$B = 6/3 = 2$

$= \frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}$

$= -e^{3t} + 2e^{t} + 3te^{t}$

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$$\frac{s-5}{s^2+4s+20} = \frac{\cancel{As+B}}{s^2+4s+20}$$

$$\frac{s-5}{s^2+4s+20} = \frac{As+B}{s^2+4s+20}$$

$s-5 = As+B$

let $s = 0$

$0-5 = A(0)+B$

$B = -5$

let $s = 1$

$s-5 = As+B$

$\cancel{s-5} = 1-5 = A(1)+B$

$-4 = A+B$

$-4 = A-5$

$-4+5 = A$

$A = 1$

$= \cancel{B} = \frac{s-5}{s^2+4s+20}$

$s^2+4s+20 = 0$

$s^2+4s = -20$

$s^2+4s+\left(\frac{4}{2}\right)^2 = -20+\left(4/2\right)^2$

$s^2+4s+(2)^2 = -20+4$

$(s+2)^2 = -16$