IBRAHIM AYODOTUN OLUWAFERANMI
15/ENG06/035

$-n(n-1)$
$-n^2+n$

1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

~~$C - 1x^2$~~

$(1-x^2)y'' - 2xy' + 2y = 0$

for sub I

$v = (1-x^2)$ $v' = -2x$ $v'' = -2$

$v''' = 0$

$u^n = y^{n+2}$

Apy leibinitz method

$u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v''$

$= y^{n+2}(1-x^2) + n y^{n+2-1} \cdot -2x +$

$\frac{n(n-1)}{2!} y^{n+2-2} \cdot -2$

$= (1-x^2) y^{n+2} - 2nxy^{n+1} - n(n-1) y^n$

for sub II

$-2xy'$

$v = -2x$ $v' = -2$ $v'' = 0$

$u^n = y^{n+1}$

$u^n v + n u^{n-1} v'$

$= y^{n+1} \cdot -2x + n y^{n+1-1} \cdot -2$

$= -2xy^{n+1} - 2ny^n$

for sub III

$v = 2$ $v' = 0$ $u^n = y^n$

$u^n v$

$= y^n \cdot 2$

$= 2y^n$

combination of sub I & II & III

$(1-x^2)y^{n+2} - 2nxy^{n+1} - n(n-1)y^n - 2xy^{n+1}$

$-2ny^n + 2y^n$

$=$

CN

$(1-x^2)y^{n+2} - 2nxy^{n+1} - 2xy^{n+1} - n(n-1)y^n -$
$2ny^n + 2y^n$

$= (1-x^2)y^{n+2} + 2xy^{n+1}(-n-1) + y^n(-n^2+n$
$-2n + 2)$

when $x = 0$

$= y^{n+2} + y^n(-n^2+n-2n+2)$

$y^{n+2} = -y^n(-n^2+n-2n+2)$

$y^{n+2} \quad y^n(n^2+n-2)$

when $n = 0$

$(y^2)_0 = -y^0(2)$

$(y^2)_0 = -2(y^0)_0$

$n = 1$

$(y^3)_0 = -y^{(1)}(0)$

$(y^3)_0 = 0$

$n = 2$

$(y^4)_0 = (y^2)_0 \cdot 4 = -8(y^0)_0$

$n = 3$

$(y^5)_0 = (y^3)_0 \cdot 10$

$(y^5)_0 = 0$

$n = 4$

$(y^6)_0 = (y^4)_0 \cdot 18 = -8(y^0) \times 18$

$(y^6)_0 = -144(y^0)_0$

$n = 5$

$(y^7)_0 = (y^5)(28)$

$(y^7)_0 = 0$

$n = 6$

$(y^8)_0 = (y^6)_0(40)$

$= 40(-144)y^0_0$

$= -5760(y^0)_0$

$y = y^0 + x(y')_0 + \frac{x^2}{2!}(y'')_0 + \frac{x^3}{3!}(y''')_0 + \frac{x^4}{4!}(y^{iv})_0$

$+ \frac{x^5}{5!}(y^v)_0 + \frac{x^6}{6!}(y^{vi})_0$

$= y^0 + x(y')_0 - (y^0)_0 + 0 - \frac{1}{3}(y^0)_0 + 0$

2i) $3e^{-4t} - 5e^{4t}$

$L\{3e^{-4t}\} = \frac{3}{s+4}$

$L[5e^{4t}] = \frac{5}{s-4}$

$\frac{3}{s+4} - \frac{5}{s-4}$

$\frac{3(s-4) - 5(s+4)}{(s+4)(s-4)}$

$\frac{3s-12-5s-20}{(s+4)(s-4)}$

$= \frac{-2s-32}{(s+4)(s-4)}$

2) $\sin 4t + \cos 4t$

$L\{\sin 4t\} = \frac{4}{s^2+16}$

$L\{\cos 4t\} = \frac{s}{s^2+16}$

$\frac{4}{s^2+16} + \frac{s}{s^2+16}$

$\frac{4+s}{s^2+16}$

$\frac{4+s}{s^2+16}$

3) $t^3 + 2t^2 - t + 4$

$L\{t^3\} = \frac{n!}{s^{n+1}}$

$= \frac{6}{s^4}$

$L\{2t^2\} = 2\frac{n!}{s^{n+1}}$

$= \frac{4}{s^3}$

$L\{t\} = \frac{n!}{s^{n+1}} = \frac{1}{s^2}$

$L\{4\} = \frac{4}{s}$

$\frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

$\frac{6+4s-s^2+4s^3}{s^4}$

$\frac{6+4s-s^2+4s^3}{s^4}$

iv) $e^{-2t}\cos 5t$

$L\{\cos 5t\} = \frac{s}{s^2+25}$

replacing s with [s+2

$= \frac{s+2}{(s+2)^2+25}$

$= \frac{s+2}{s^2+4s+4+25}$

$= \frac{s+2}{s^2+4s+29}$

v) $t \sin 3t$

$L\{\sin 3t\} = \frac{a}{s^2+a^2}$

$= \frac{3}{s^2+9}$

$-F'(s) = -\frac{d}{ds}\left[\frac{3}{s^2+9}\right]$

Applying quotient rule

$\frac{V\,du/dv - U\,dv/ds}{V^2}$

$U = 3 \quad dU = 0 \quad V = s^2+9 \quad dV = 2s$

$\frac{s^2+9(0) - 3(2s)}{(s^2+9)^2}$

vii) $[e^{4t} \cos 2t]$

$\cos 2t = \left[\frac{s}{s^2+4}\right]$

$e^{4t} = [s-4]$

$= \left[\frac{[s-4]}{[s-4]^2+4}\right]$

viii) $[t \sin 2t]$

$\sin 2t \left[\frac{2}{s^2+4}\right]$

$-\frac{d}{ds}\left[\frac{2}{s^2+4}\right] \begin{matrix} u \\ v \end{matrix}$

Applying quotient rule

$U = 2 \quad du = 0 \quad V = s^2+4$

$dV = 2s$

$-\left[\frac{V\, du/ds - U\, dv/ds}{V^2}\right]$

$-\left[\frac{0 - 2 \times 2s}{[s^2+4]^2}\right]$

$\frac{4s}{[s^2+4]^2}$

ix) $t^2 \cos t$

$\cos t = \frac{s}{s^2+a^2}$

$t\cos t = -\frac{d}{ds}\left[\frac{s}{s^2+a^2}\right]$

$U = s \quad du = 1$

$V = s^2+1 = dV = 2s$

$\frac{V\, du/ds - U\, dV/ds}{V^2}$

$\left(\frac{(s^2+1)(1) - (s)(2s)}{(s^2+1)^2}\right)$

$= \left(\frac{s^2+1-2s^2}{(s^2+1)^2}\right)$

$L^{-1}[t\cos t] = \cancel{\left(\frac{s^2+1-2s^2}{(s^2+1)^2}\right)}$

$= \frac{-s^2+1}{(s^2+1)^2} = \frac{-1(s^2+1)}{(s^2+1)^4} = \frac{-1}{s^2+1}$

$L[t^2\cos t] = -\frac{d}{ds}\left[\frac{1}{s^2+1}\right]$

$U = 1 \quad du/ds = 0$

$V = s^2+1 \quad dv/ds = 2s$

$\frac{(s^2+1)(0) - (1)(2s)}{(s^2+1)^2}$

$= \frac{-2s}{(s^2+1)^2}$

3) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{(s-3)} + \frac{B}{s-4}$

$s-5 \quad \frac{A(s-4) + B(s-3)}{(s-3)(s-4)}$

$A(s-4) + B(s-3) = s-5$

at $s=4$

$B(1) = -1$

$B = -1$

at $s = 3$

$A(-1) + 0 = -2$

$-A = -2$

$A = 2$

ANS $= 2e^{3t} - e^{4t}$

ii) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$A(s-4) + B(s-2) = 2s-4$

At $s=4$

$0 + B(2) = 2B$

$B = 1$

at $s=2$

$A(2-4)+0=2(2)-6=4-6=-2$

$-2A=-2$

$A=1$

$=e^{2t}+e^{4t}$

iii $\frac{5s-8}{s(s-4)}=\frac{A}{s}+\frac{B}{s-4}$

$A(s-4)+Bs=5s-8$

at $s=0$

$A(-4)+0=-8$

$A=2$

at $s=4$

$4B=12$

$B=3$

$2+3e^{-4t}$

iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2}=\frac{A}{s-3}+\frac{B}{s-1}+\frac{C}{(s-1)^2}$

$A(s-1)^2+B(s-3)(s-1)+C(s-3)=s^2-3s-4$

at $s=1$

$-2C=-6$

$C=3$

at $s=3$

$4A=-4$

$A=-1$

$A+B=-1$

$-1+B=-1$

$B+1-(-1)$

$B=2$

$-e^{3t}+2e^{t}+3te^{t}$

V $\frac{s-5}{s^2+4s+20}=\frac{s-5}{(s+2)^2+4^2}$

$=\frac{A}{(s+2)}+\frac{B}{(s+2)^2}+\frac{C}{16}$

$=A(s+2)(16)+B(16)+C(s+2)^2=5s$

s^2C

$\frac{s-5}{s^2+4s+4+16}=\frac{s-5}{(s+2)^2+16}$

$\frac{s}{(s+2)^2+16}-\frac{5}{(s+2)^2+16}$

$\frac{s+2-2}{(s+2)^2+16}-\frac{5}{(s+2)^2+16}$

$\frac{s+2}{(s+2)^2+16}-\frac{2}{(s+2)^2+16}-\frac{5}{(s+2)^2+16}$

$\frac{s+2}{(s+2)^2+4^2}-\frac{7\times 4}{(s+2)^2+4^2}$

$e^{-2t}\cos 4t-\frac{7}{4}e^{-2t}\sin 4t$