1) Ebx

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1 $(1-x^2)\,{}^{dy}/_{dx} - 2x\,{}^{dy}/_{dx} + 2y = 0$

$(1-x^2)y^2 - 2xy' + 2y = 0$

$y^n = U^n V + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^2 + \cdots$

$[y^{(2+n)} \cdot (1-x^2) + ny^{(1+n)} \cdot (-2x) + \frac{n(n-1)y^n}{2!} \cdot (-2)] + [y^{(1+n)} \cdot -2x$

$+ny^n \cdot -2 \;+ [2y^n] = 0$

$(1-x^2)y^{(2+n)} - 2xny^{(1+n)} - n(n-1)y^{(n)} - 2xy^{(1+n)} - 2ny^{(n)} + 2y^{(n)} = 0$

let $x = 0$

$y^{(2+n)} - n(n-1)y^{(n)} - 2ny^{(n)} + 2y^n = 0$

$y^{n+2} + y^n[-n(n-1) - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 + n - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 - n + 2] = 0$

$y^{n+2} = -(y^n)_0[-n^2 - n + 2]$

$n = 0$:- $y^2 = -y^0 \cdot 2 = -2y^0$

$n = 1$ $y^3 = -y' \cdot [0] = 0$

$n = 2$:- $y^4 = -y^2[-4] = 4y^2 = 4(-2y^0) = -8y^0$

$n = 3$:- $y^5 = -y^3 \cdot [-10] = 10y^3 = 10 \cdot 0 = 0$

$n = 4$:- $y^6 = -y^4 \cdot [-18] = 18y^4 = 18 \cdot y = -2y^0$

$n = 5$:- $y^7 = -y^5 \cdot [-28] = 28(y^5)_0 = 28 \cdot 0 = 0$

$y = y^0 + xy' + \frac{x^2}{2!}y^3 + \frac{x^3}{3!}y^3 + \cdots$

$y = y^0 + xy' + \frac{x^2}{2!}(-2)y^0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)y^0 + \cdots$

$+ \frac{x^5}{5!}(0) + \frac{x^6}{6!}(18) + (-2)y^0 + \frac{x^7}{7!}(0)$

$y = y^0 + xy' - x^2y^0 - \frac{x^4}{3 \times 1}y^0 - \frac{x^6}{5}y^0$

$y = y^0[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}] + y'_0[x]$

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2) Transform each of the following function into laplace(s) domain

i) $3e^{-4t} - 5e^{4t}$

Recall $L\{F(t) \pm G(t)\} = L\{F(t)\} \pm L\{G(t)\}$.

$= L\{3e^{-4t} - 5e^{4t}\} \Rightarrow L\{3e^{-4t}\} - L\{5e^{4t}\} \Rightarrow 3L\{e^{-4t}\} - 5L\{e^{4t}\}$

$$= 3 \cdot \left\{\frac{1}{s-a}\right\} - 5 \cdot \left\{\frac{1}{s-a}\right\}$$

$$= 3 \cdot \left\{\frac{1}{s-(-4)}\right\} - 5\left\{\frac{1}{s-4}\right\}$$

$$= \frac{3}{s+4} - \frac{5}{s-4}$$

ii) $\sin 4t + \cos 4t$

$\Rightarrow L\{\sin 4t + \cos 4t\} = L\{\sin 4t\} + L\{\cos 4t\}$

$$= \frac{a}{s^2+a^2} + \frac{s}{s^2+a^2}$$

$$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$$

$$= \frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{(s^2+16)}$$

iii) $t^3 + 2t^2 - t + 4$

$\Rightarrow L\{t^3+2t^2-t+4\} = L\{t^3\} + L\{2t^2\} - L\{t\} + L\{4\}$

Recall,

$$t^n = \frac{n!}{s^{n+1}}$$

$$= \frac{3!}{s^{3+1}} + 2\left\{\frac{2!}{s^{2+1}}\right\} - \left\{\frac{1!}{s^{1+1}}\right\} + \frac{4}{s}$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

iv) $e^{-2t}\cos 5t$

Recall the first shift theorem

$$L\{\cos 5t\} = \frac{s}{s^2+a^2}$$

$$= \frac{s}{s^2+5^2} = \frac{s}{s^2+25}$$

replacing s by a shift of e^{-2t} :- $s+2$

$$\therefore L\{e^{-2t}\cos 5t\} = \frac{s+2}{[s+2]^2+25}$$

V) $t \sin 3t$

$$L\{\sin 3t\} = \frac{a}{s^2+a^2}$$

$$= \frac{3}{s^2+3^2}$$

$$F(s) = \frac{3}{s^2+9}$$

$$L\{t\sin 3t\} = -F'(s)$$

$F'(s)$ = using quotient rule.

$u = 3$ $\quad du/ds = 0$

$v = s^2+9$ $\quad dv/ds = 2s$

$$\therefore \frac{v\,du/ds - u\,dv/ds}{v^2}$$

$$= \frac{[s^2+9]\cdot 0 - 3\cdot[2s]}{[s^2+9]^2}$$

$$= \frac{-6s}{[s^2+9]^2}$$

$$\therefore -F'(s) = -1\cdot\left\{\frac{-6s}{[s^2+9]^2}\right\}$$

$$= \frac{6s}{[s^2+9]^2}$$

Vi) $\frac{e^{-t} - e^{-2t}}{t} =$

$$L\left\{\frac{e^{-t}-e^{-2t}}{t}\right\}$$

$$\lim_{t\to 0}\left\{\frac{e^{-t}-e^{-2t}}{t}\right\} = \frac{1-1}{0} = \frac{0}{0} \Rightarrow \text{Indeterminate}$$

Applying L'Hospital's rule.

$$\lim_{t \to 0} \left\{ \frac{-1e^{-t} - (-2)e^{-2t}}{1} \right\} = \left\{ \frac{-1 + 2}{1} \right\} = \frac{1}{1} = 1 \quad \{\text{determinate}\}$$

$$L\left\{ \frac{f(t)}{t} \right\} = \int_{\sigma = s}^{\infty} F\sigma \, d\sigma$$

$$F\sigma = L\{F(t)\}$$

$$L\{f(t)\} = e^{-t} - e^{-2t} = \frac{1}{s+a} - \frac{1}{s+a}$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$F\sigma = \frac{1}{\sigma+1} - \frac{1}{\sigma+2}$$

$$\int F\sigma \quad L\left\{ \frac{f(t)}{t} \right\} = \int_{\sigma = s}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+2} \, d\sigma$$

$$= \int_{\sigma = s}^{\infty} \frac{1}{\sigma+1} \, d\sigma - \int_{\sigma = s}^{\infty} \frac{1}{\sigma+2} \, d\sigma$$

$$= \left[\ln(\sigma+1) - \ln(\sigma+2) \right]_s^{\infty}$$

$$= \left[\ln(\sigma+1) - \ln(\sigma+2) \right]_s^{\infty}$$

$$= \left[\ln \frac{\sigma+1}{(\sigma+2)} \right]_s^{\infty} = \ln \left[\frac{\infty+1}{(\infty+2)} - \frac{s+1}{(s+2)} \right]$$

$$= -\ln \left[\frac{s+1}{(s+2)} \right] = \ln \left[\frac{(s+2)}{s+1} \right]$$

vii) $e^{4t}\cos 2t$

$$L\{e^{4t}\cos 2t\} = e^{4t} L\{\cos 2t\}$$

$$L\{\cos 2t\} = \frac{s}{s^2+2^2}$$

$$= \frac{s}{s^2+4}$$

replacing s by a shift of e^{4t} :- $s-4$

$$L\{e^{4t}\cos 2t\} = \frac{s-4}{[s-4]^2+4}$$

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viii) $t\sin 2t$

$$= L\{t\sin 2t\} = -\frac{d}{ds}\{F(s)\}$$

$$F(s) = L\{\sin 2t\} = \frac{2}{s^2+2^2}$$

$$~~F(s)~~ = \frac{2}{s^2+4}$$

$F'(s)$ = using quotient rule.

$u = 2$ $\qquad du/ds = 0$

$v = s^2+4$ $\qquad dv/ds = 2s$

$$\frac{v\,du/ds - u\,dv/ds}{v^2}$$

$$= \frac{(s^2+4)\cdot 0 - 2\cdot(2s)}{[s^2+4]^2}$$

$$= \frac{-4s}{[s^2+4]^2}$$

$\therefore L\{t\sin 2t\} = -f'(s)$

$$= -1\cdot\left\{\frac{-4s}{[s^2+4]^2}\right\}$$

$$= \frac{4s}{\ }$$

Assignment 4 (cont'd)

ix) $t^3 + 4t^2 + 5$

$$L\{t^3+4t^2+5\} = L\{t^3\} + 4L\{t^2\} + L\{5\}$$

$$= \frac{3!}{S^{3+1}} + 4\left\{\frac{2!}{S^{2+1}}\right\} + \frac{5}{S}$$

$$= \frac{6}{S^4} + \frac{8}{S^3} + \frac{5}{S}$$

x) $e^{3t}(t^2+4)$

$\Rightarrow$ Let $x = t^2+4$

$\therefore L\{e^{3t}x\}$

$$\therefore L\{x\} = L\{t^2+4\}$$

$$= L\{t^2\} + L\{4\}$$

$$= \frac{2!}{S^{2+1}} + \frac{4}{S}$$

$$= \frac{2}{S^3} + \frac{4}{S}$$

replacing S by a shift of $S-3$

$$L\{e^{3t}x\} = \frac{2}{[S-3]^3} + \frac{4}{[S-3]}$$

xi) $t^2 \cos t$.

$$L\{t^2\cos t\} = t^2 L\{\cos t\}$$

$$F(s) = L\{\cos t\} = \frac{S}{S^2+1^2}$$

$$F(S) = \frac{S}{S^2+1^2}$$

$F'(s)$ = using quotient rule.

$u = S$ $\qquad du/ds = 1$

$V = S^2+1^2$ $\qquad dV/ds = 2S$

$$\frac{V\,du/ds - U\,dV/ds}{V^2}$$

$$= \frac{[s^2+1^2]\cdot 1 - 2s[s]}{[s^2+1^2]^2}$$

$$= \frac{s^2+1^2 - 2s^2}{[s^2+1^2]^2}$$

$$= \frac{-s^2+1}{[s^2+1^2]^2}$$

Recall

$$-f''(s) = -\frac{d}{ds}\left\{\frac{s^2-1}{[s^2+1^2]^2}\right.$$

Using quotient rule:

$$\frac{V\,^{du}/_{ds} - U\,^{dv}/_{ds}}{V^2}$$

$u = s^2-1$ $\qquad$ $^{du}/_{ds} = 2s$

$v = [s^2+1]^2$ $\qquad$ $^{dv}/_{ds} = 4s[s^2+1]$

$$\frac{[s^2+1]^2\cdot 2s - [s^2-1][4s^3+4s]}{[[s^2+1]^2]}$$

$$= \frac{[2s^5 - 4s^3+2s] - [4s^5-4s]}{[(s^2+1)^2]}$$

$$= \frac{2s^5-4s^3+2s-4s^5+4s}{[(s^2+1)^2]}$$

$$= \frac{-2s^5-4s^3+6s}{[(s^2+1)^2]}$$

$$= \frac{-2s^5-4s^3+6s}{s^4+2s^2+1}$$

$$f''(s) = \frac{-d}{ds}\cdot\left\{\frac{-2s^5-4s^3+6s}{s^4+2s^2+1}\right\}$$

$$f''(s) = \frac{2s^5+4s^3-6s}{s^4+2s^2+1}$$

B) Convert the following function to time domains.

i) $\frac{S-5}{(S-3)(S-4)}$

$$L^{-1}\left\{\frac{S-5}{(S-3)(S-4)}\right\} = \frac{A}{(S-3)} + \frac{B}{(S-4)}$$

$$\frac{S-5}{(S-3)(S-4)} = \frac{A(S-4) + B(S-3)}{(S-3)(S-4)}$$

Assuming $S=4$.

$4-5 = A(4-4) + B(4-3)$

$-1 = B(1)$

$B = -1$

Assuming $S=3$

$3-5 = A(3-4) + B(3-3)$

$-2 = A(-1)$

$A = 2$

$$\therefore \frac{S-5}{(S-3)(S-4)} = \frac{2}{(S-3)} + \frac{-1}{(S-4)}$$

$$= \frac{2}{S-3} - \frac{1}{S-4}$$

$$= 2\left\{\frac{1}{S-3}\right\} - \left\{\frac{1}{S-4}\right\}$$

$$= 2e^{3t} - e^{4t}$$

ii) $\frac{2S-6}{(S-2)(S-4)}$

$$L^{-1}\left\{\frac{2S-6}{(S-2)(S-4)}\right\}$$

$-n^2y$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$$

$2s-6 = A(s-4) + B(s-2)$

Assuming $s=4$

$(2(4)-6) = A(4-4) + B(4-2)$

$8-6 = A(0) + B(2)$

$2 = 2B$

$B = 1$

Assuming $s=2$

$2(2)-6 = A(2-4) + B(2-2)$

$4-6 = -2A + 0$

$-2 = -2A$

$A = 1$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

$$L^{-1}\left\{\frac{2s-6}{(s-2)(s-4)}\right\} = e^{2t} + e^{4t}$$

iii) $\frac{5s-8}{s(s-4)}$

$$L^{-1}\left\{\frac{5s-8}{s(s-4)}\right\} = \frac{A}{s} + \frac{B}{s-4}$$

$5s-8 = A(s-4) + B(s)$

Assuming $s=4$

$5(4)-8 = A(4-4) + B(4)$

$20-8 = 4B$

$12 = 4B$

$B = 3$

Assuming $s=0$

$5(0)-8 = A(0-4) + B(0)$

$-8 = -4A$

$A = 2$

$$L^{-1}\left\{\frac{s2-8}{s(s-4)}\right\} = \frac{2}{s} + \frac{3}{s-4}$$

$$= \frac{2}{s} + 3\left\{\frac{1}{s-4}\right\}$$

$$= 2 + 3e^{4t}$$

iv) $\frac{s-5}{s^2+4s+20}$

$$L^{-1}\left\{\frac{s-5}{s^2+4s+20}\right\} =$$

$$F(s) = \frac{s-5}{s^2+4s+4+16} = \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16} = \frac{s+2}{(s+2)^2+16} - \frac{2}{(s+2)^2+16}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7 \times 4/4}{(s+2)^2+4^2}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \cdot \frac{4}{(s+2)^2+4^2}$$

$$F(t) = e^{-2t}\cos 4t - 7/4\, e^{-2t}\sin 4t$$

$$= e^{-2t}\left[\cos 4t - 7/4 \sin 4t\right]$$

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iv $\dfrac{s^2-3s-4}{(s-3)(s-1)^2}$

$$= f(s) = \frac{A}{s+3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$$

$$A: \left.\frac{s^2-3s-4}{(s-1)^2}\right|_{s=3} = \frac{3^2-3(3)-4}{(3-1)^2} = -1.$$

$$B: \left.\frac{s^2-3s-4}{s-3}\right|_{s=1} = \frac{1^2-3(1)-4}{1-3} = 3$$

$$C: \frac{d}{ds}\left[\frac{s^2-3s-4}{s-3}\right]_{s=1} = \frac{(s-3)(2s-3)-[s^2-3s-4]}{(s-3)^2}$$

at $s=1$

$$\frac{(1-3)(2(1)-3)-[1^2-3(1)-4]}{(1-3)^2} = 2$$

$$f(s) = \frac{-1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}$$

$$f(t) = -e^{-3t} + 3te^{t} + 2e^{t}$$

$$= e^{t}[3t+2] - e^{3t}$$