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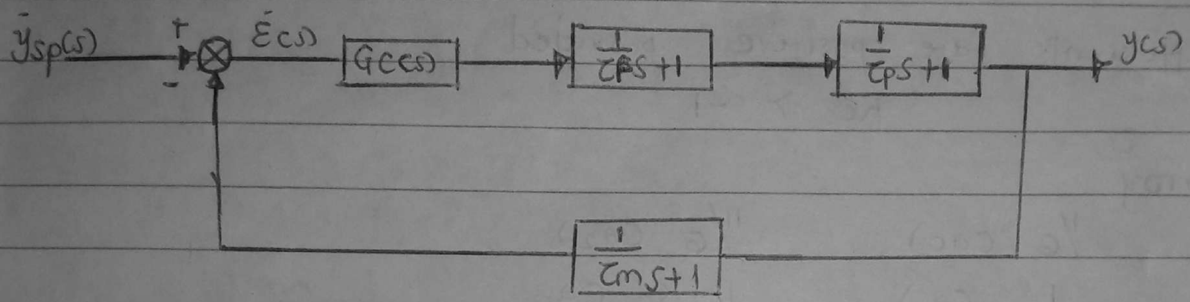
DEPT: CHEMICAL ENGINEERING

CHE 531 ASSIGNMENT IV.



P only controller : $G_c(s) = K_c$

$\tau_p = 1$, $\tau_f = 1/2$, $\tau_m = 1/3$

$$G_f = \frac{1}{\tau_f s+1} = \frac{1}{1/2 s+1}$$

$$G_p = \frac{1}{\tau_p s+1} = \frac{1}{s+1}$$

$$G_m = \frac{1}{\tau_m s+1} = \frac{1}{1/3 s+1}$$

Characteristic equation

$$1 + G_p G_c G_f G_m = 0$$

Hence,

$$1 + \frac{1}{s+1} \cdot K_c \cdot \frac{1}{1/2 s+1} \cdot \frac{1}{1/3 s+1} = 0$$

$$1 + \frac{K_c}{(s+1)(1/2 s+1)(1/3 s+1)} = 0$$

$$(s+1)(1/2 s+1)(1/3 s+1) = (1/2 s^2 + s + 1/2 s + 1)(1/3 s+1)$$

$$= (1/2 s^2 + 3/2 s + 1)(1/3 s + 1)$$

$$= 1/6 s^3 + 1/2 s^2 + 1/3 s + 1/2 s^2 + 3/2 s + 1$$

$$= 1/6 s^3 + s^2 + 11/6 s + 1$$

$$1 + \frac{K_c}{1/6 s^3 + s^2 + 11/6 s + 1} = 0$$

$$\frac{\frac{1}{6}s^3 + s^2 + \frac{11}{6}s + 1 + K_c}{\frac{1}{6}s^3 + s^2 + \frac{11}{6}s + 1} = 0$$

$$\frac{1}{6}s^3 + s^2 + \frac{11}{6}s + 1 + K_c = 0$$

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} \quad a_3 s^{n-3}$$

All coefficients are positive provided

$$K_c > -1$$

Routh array

Row 1	$\frac{1}{6}$ (a_0)	$\frac{11}{6}$ (a_2)
Row 2	1 (a_1)	$1+K_c$ $(a_3$
Row 3	$\frac{5}{3} - \frac{1}{6}K_c$ (A_1)	0 (A_2)
Row 4	$1+K_c$ (B_1)	

$$A_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} = \frac{1 \times \frac{11}{6} - \frac{1}{6}(1+K_c)}{1} = \frac{11}{6} - \frac{1}{6} - \frac{1}{6}K_c$$

$$= \frac{5}{3} - \frac{1}{6}K_c.$$

$$A_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} = \frac{1 \times 0 - \frac{1}{6} \times 0}{1} = 0$$

$$B_1 = \frac{A_1 a_3 - A_2 a_1}{A_1} = \frac{\cancel{\left(\frac{5}{3} - \frac{1}{6}K_c\right)}(1+K_c)}{\cancel{\left(\frac{5}{3} - \frac{1}{6}K_c\right)}}$$

$$B_1 = 1 + K_c$$

To have a stable system, each element in the left column of the routh array must be positive. Element A_1 will be positive if

$K_c > \left(\frac{5}{3} + \frac{1}{6}\right) = 10$

Similarly B will be positive if $K_c > -1$

Thus, it can be concluded that the system will be stable if

$-1 < K_c < 10$

2. PI controller $G_c(s) = K_c + \frac{K_c}{\tau_I s} = 5 + \frac{5}{0.25s} = \frac{5/4\,s + 5}{0.25s}$

$G_f = \frac{1}{1/2\,s+1}$ $G_p = \frac{1}{s+1}$ $G_m = \frac{1}{1/3\,s+1}$

$1 + G_p G_f G_c G_m = 0$

$1 + \left(\frac{1}{s+1}\right)\left(\frac{1}{1/2\,s+1}\right)\left(\frac{5/4\,s+5}{1/4\,s}\right)\left(\frac{1}{1/3\,s+1}\right) = 0$

$1 + \frac{5/4\,s + 5}{(s+1)(1/2\,s+1)(1/4\,s)(1/3\,s+1)} = 0$

from question 1 $(s+1)(1/2\,s+1)(1/3\,s+1) = 1/6\,s^3 + s^2 + 11/6\,s + 1$

$(s+1)(1/2\,s+1)(1/3\,s+1)(1/4\,s) = (1/6\,s^3 + s^2 + 11/6\,s + 1)(1/4\,s)$

$= 1/24\,s^4 + 1/4\,s^3 + 11/24\,s^2 + 1/4\,s$

$1 + \frac{5/4\,s + 5}{1/24\,s^4 + 1/4\,s^2 + 11/24\,s^2 + 1/4\,s} = 0$

$\frac{1/24\,s^4 + 1/4\,s^3 + 11/24\,s^2 + 1/4\,s + 5/4\,s + 5}{1/24\,s^4 + 1/4\,s^3 + 11/24\,s^2 + 1/4\,s} = 0$

$\frac{1/24\,s^4 + 1/4\,s^3 + 11/24\,s^2 + 3/2\,s + 5}{1/24\,s^4 + 1/4\,s^3 + 11/24\,s^2 + 1/4\,s} = 0$

The characteristic equation is $1/24\,s^4 + 1/4\,s^3 + 11/24\,s^2 + 3/2\,s + 5 = 0$

$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3}\ a_4 s^{n-4}$

Row 1	$1/24\ (a_0)$	$11/24\ (a_2)$	$5\ (a_4)$
Row 2	$1/4\ (a_1)$	$3/2\ (a_3)$	$0\ (a_5)$
Row 3	$5/24\ (A_1)$	$5\ (A_2)$	
Row 4	$3/2\ (B_1)$		
Row 5	$5\ (C_1)$		

$A_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} = \frac{1/4 \times 11/24 - 1/24 \times 3/2}{1/4} = 5/24$

$A_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} = \frac{1/4 \times 5 - 1/24 \times 0}{1/4} = 5$

$A_3 = 0$

$$B_1 = \frac{A_1 a_3 - A_2 a_1}{A_1} = \frac{5/24 \times 3/2 - 5 \times 1/4}{5/24} = -9/2$$

$$B_2 = \frac{A_1 a_5 - a_1 A_3}{A_1} = 0$$

$$C_1 = \frac{B_1 A_2 - A_1 B_2}{B_1} = \frac{-9/2 \times 5 - 5/24 \times 0}{-9/2} = 5$$

All the coefficient in the first column are not positive therefore the system is unstable.

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