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15/ENG06/013

MECHANICAL ENGINEERING

1. $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ using Leibnitz-Maclaurin Series

Solution

$(1-x^2)y'' - 2xy' + 2y = 0$

Sub 1

$(1-x^3)y''$

$u = y'' \therefore u^n = y^{n+2}$

$V = 1-x^2$

$V' = -2x$

$V'' = -2$

$\therefore U^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \cdots$

$y^n = y^{n+2}(1-x^2) + n y^{n+1} \cdot -2x + \frac{n(n-1)}{2!} y^n \cdot -2 + \cdots$

$y^n = (1-x^2)y^{n+2} - 2nxy^{n+1} - n(n-1)y^n$

Sub 2

$-2xy'$

$u = y' \quad u^n = y^{n+1}$

$V = -2x, \quad V' = -2,$

Using $U^n V + n u^{n-1} V' + \frac{n(n-1)}{2!} U^{n-2} V^2 + \cdots$

$y^{n+1} \cdot -2x + n y^n \cdot -2$

$= -2xy^{n+1} - 2ny^n$

Sub 3

$u = 2y$

$u = y. \quad u^n = y^n$

$V = 2, \quad V' = 0$

$y^n =$ Using $U^n v + n u^{n-1} v' + \cdots$

$y^n \cdot 2 = 2y^n$

Combining Sub ①, ② and ③

$(1-x^2)y^{n+2} - 2nxy^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n$

$(1-x^2)y^{n+2} + (-2nx - 2x)y^{n+1} + (-n^2 + n - 2n + 2)y^n = 0$

$(1-x^2)y^{n+2} - 2nx + 2xy^{n+1} - (n^2 - n + 2)y^n = 0$

at $x = 0$

$y^{(n+2)} - 0(-n^2 - n + 2)y^n = 0$

$y^{(n+2)} = (y^n)_0(n^2 + n - 2)$

$\therefore$ when $n = 0$

$(y^2)_0 = (y^0)_0(-2) = -2(y^0)_0$

when $n = 1$

$(y^3)_0 = (y^1)_0(0) = 0$

when $n = 2$

$(y^4)_0 = (y^2)_0(4) = 4(-2)(y^0)_0 = -8(y^0)_0$

when $n = 3$

$(y^5)_0 = (y^3)_0(10) = 10(y^3)_0 = 10 \times 0 = 0$

when $n = 4$

$(y^6)_0 = (y^4)_0(18) = 18(y^4)_0 = 18 \times -8(y^0)_0 = -144(y^0)_0$

when $n = 5$

$(y^7)_0 = (y^5)_0(28) = 28 \times 0 = 0$

when $n = 6$

$(y^8)_0 = (y^6)_0(40) = 40 \times -144(y^{(0)})_0 = -5760(y^0)_0$

$\therefore y = y_0 + x(y^1)_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \cdots$

$y = y_0 + x(y^1)_0 + (-2y^0)_0 \frac{x^2}{2!} + 0 + (-8y^0)\frac{x^4}{4!} + 0 + \frac{x^6}{6!}(-144)(y^0)_0$

$+ 0 + (-5760)(y^0)_0 \frac{x^8}{8!}$

$y = y_0\left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} - \frac{x^8}{7}\right] + (y^1)_0(x)$

Questions 2

2 i) $L\{3e^{-4t} - 5e^{4t}\}$

$$= \frac{3}{s+4} - \frac{5}{s-4} = \frac{3(s-4) - 5(s+4)}{(s+4)(s-4)}$$

$$= \frac{3s - 12 - 5s - 20}{s^2 - 16} = \frac{-2s - 32}{s^2 - 16}$$

ii) $L\{\sin 4t + \cos 4t\}$

$$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2} = \frac{4+s}{s^2+16}$$

iii) $L\{t^3 + 2t^2 - t + 4\}$

$$= \frac{3!}{s^4} + 2\left(\frac{2!}{s^3}\right) - \frac{1}{s^2} + \frac{4}{s}$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s} = \frac{6 + 4s - s^2 + 4s^3}{s^4}$$

iv) $L\{e^{-2t} \cos 5t\}$

$$L\{\cos 5t\} = \frac{s}{s^2+25}$$

Let $s = s+2$

$$L\{e^{-2t} \cos 5t\} = \frac{s+2}{(s+2)^2 + 25} = \frac{s+2}{s^2 + 4s + 29}$$

v) $L\{t \sin 3t\}$

$$L\{\sin 3t\} = \frac{3}{s^2+3^2} = \frac{3}{s^2+9}$$

$$-F'(s) = -\frac{d}{ds}\left[\frac{3}{s^2+9}\right]$$

$$\frac{s^2+9(0) - 3(2s)}{(s^2+9)^2} = \frac{-6s}{(s^2+9)^2}$$

vi) $L\left\{\frac{e^{-t} - e^{-2t}}{t}\right\}$

$$\lim_{t \to 0} \frac{e^{-t} - e^{-2t}}{t} = \frac{e^{-0} - e^{-0}}{0} = \frac{1-1}{0} = \frac{0}{0}$$

using L'hospital rule

$$L\left\{\frac{-e^{-t} + 2e^{-2t}}{1}\right\} = \frac{-1+2}{1} = \frac{1}{1}$$

vii) $L\{e^{4t} \cos 2t\}$

$$L\{\cos 2t\} = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

Let $s = s-4$

$$L\{e^{4t} \cos 2t\} = \frac{s-4}{(s-4)^2+4} = \frac{s-4}{s^2 - 8s - 12}$$

viii) $L\{t \sin 2t\}$

$$L\{\sin 2t\} = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$$

$$L\{t \sin 2t\} = -F'(s) = -\frac{d}{ds}\left[\frac{2}{s^2+4}\right]$$

$$\frac{s^2+4(0) - 2(2s)}{(s^2+4)^2} = \frac{-4s}{(s^2+4)^2}$$

ix) $L\{t^3 + 4t^2 + 5\}$

$$= \frac{3!}{s^4} + \frac{4 \cdot 2!}{s^3} + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{2} = \frac{6 + 8s + 5s^3}{s^4}$$

x) $L\{e^{3t}(t^2+4)\}$

$$L\{t^2 e^{3t}\} + L\{4e^{3t}\}$$

$$L\{t^2\} = \frac{2!}{s^3}$$

Let $s = s-3$

$$L\{t^2 e^{3t}\} = \frac{2}{(s-3)^3}$$

$$L\{e^{3t}(t^2+4)\} = \frac{2}{(s-3)^2} + \frac{4}{s-3}$$

$s^2 y_{(s)} - 2s - 1 - 2s y_{(s)} - 2 + 5y_{(s)} = \frac{1}{s-2}$

2.

$= \frac{2+4s^2-24s+36}{(s-3)^3} = \frac{4s^2-24s+38}{(s-3)^3}$

$L\{t^2 \cos t\}$

$L\{\cos t\} = \frac{s}{s^2+1^2} = \frac{s}{s^2+1}$

$-F'(s) = \frac{-d}{ds}\left[\frac{s}{s^2+1}\right]$

$\frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2} = \frac{s^2+1-2s^2}{((s^2+1))^2}$

$= \frac{-s^2+1}{(s^2+1)^2} = \frac{-1}{s^2+1}$

$L\{t^2 \cos t\} = \frac{-d}{ds}\left[\frac{-1}{s^2+1}\right]$

$\frac{(s^2+1)(0) - 1(2s)}{(s^2+1)^2} = \frac{-2s}{(s^2+1)^2}$

Question 3

$\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$A(s-4) + B(s-3) = s-5$

if $s=4$

$B(1) = -1$

$B = -1$

if $s=3$

$-A = -2$

$A = 2$

$\therefore \frac{A}{s-3} + \frac{B}{s-4} = \frac{2}{s-3} - \frac{B}{s-4}$

$= 2e^{3t} - e^{4t}$

$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$A(s-4) + B(s-2) = 2s-6$

if $s=4$

$B(2) = 2(4) - 6 = 2$

$B = 1$

if $s=2$

$A(2-4) = 2(2) - 6 =$

$-2A = -2$

$A = 1$

$\Rightarrow \frac{1}{s-2} + \frac{1}{s-4} = e^{2t} + e^{4t}$

iii) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$A(s-4) + B(s) = 5s-8$

at $s=0$

$-4A = -8$

$A = 2$

at $s=4$

$4B = 12$

$B = 3$

$\Rightarrow \frac{2}{s} + \frac{3}{s-4} = 2 + 3e^{4t}$

iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$

$A(s-1)^2 + B(s-3)(s-1) + C(s-3) = s^2-3s-4$

at $s=1$

$0 + 0 + C(-2) = 1^2 - 3(1) - 4$

$-2C = -6$

$C = 3$

at $s=3$

$A(2)^2 = 3^2 - 3(3) - 4$

$4A = -4$

$A = -1$

$A + B = 1$

$B = 1 - A = 1 - (-1) = 1 + 1 = 2$

2.)

$$\Rightarrow \frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}$$

$$= -e^{3t} + 2e^{t} + 3te^{t}$$

v. $\frac{s-5}{s^2+4s+20} = \frac{s-5}{(s+2)^2+4^2}$

ii) $= \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{16}$

$= A(s+2)(16) + B(16) + C(s+2)^2 = s-5$

$= (16s+32)A + 16B + (s^2+4s+4)C = s-5$

iii) Comparing coefficients

$C = 0$

$16A + 4C = 1$

$32A + 16B + 4C = -5$

$\therefore C = 0$

$16A + 4C = 1$

iv) $16A + 4(0) = 1$

$A = 1/16$

$32A + 16B + 4C = -5$

$32\left(\frac{1}{16}\right) + 16B + 4(0) = -5$

$2 + 16B = -5$

$16B = -5-2 = -7$

$B = \frac{-7}{16}$

v) $\Rightarrow \frac{1}{16(s+2)} - \frac{7}{16(s+2)^2} + 0$

$= \frac{1}{16} e^{-2t} - \frac{7}{16} te^{-2t}$

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