3

(3i) $\frac{s-5}{(s-3)(s-4)}$

$$L^{-1}\left[\frac{s-5}{(s-3)(s-4)}\right] = \frac{A}{(s-3)} + \frac{B}{(s-4)}$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A(s-4)+B(s-3)}{(s-3)(s-4)}$$

Assuming $s=4$

$$4-5 = A(4-4)+B(4-3)$$

$$-1 = B(1)$$

$$B = -1$$

Assuming $s=3$

$$3-5 = A(3-4)+B(3-3)$$

$$-2 = A(-1)$$

$$A = 2$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} + \frac{-1}{s-4}$$

$$= \frac{2}{s-3} - \frac{1}{s-4}$$

$$= 2\left[\frac{1}{s-3}\right] - \left[\frac{1}{s-4}\right]$$

$$= 2e^{3t} - e^{4t}$$

ii) $\frac{2s-6}{(s-2)(s-4)}$

$$L^{-1}\left[\frac{2s-6}{(s-2)(s-4)}\right]$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$$

$$2s-6 = A(s-4)+B(s-2)$$

Assuming $s=4$

$2(4)-6 = A(4-4) + B(4-2)$

$8-6 = A(0) + B(2)$

$2 = 2B$

$B = 1$

$S = 2$

$2(2)-6 = A(2-4) + B(2-2)$

$4-6 = -2A + 0$

$-2 = -2A$

$A = 1$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

$$L^{-1}\left[\frac{2s-6}{(s-2)(s-4)}\right] = e^{2t} + e^{4t}$$

(iii) $\frac{5s-8}{s(s-4)}$

$$L^{-1}\left[\frac{5s-8}{s(s-4)}\right] = \frac{A}{s} + \frac{B}{s-4}$$

$5s - 8 = A(s-4) + B(s)$

Assuming $S = 4$

$5(4) - 8 = A(4-4) + B(4)$

$20 - 8 = 4B$

$12 = 4B$

$B = 3$

Assuming $S = 0$

$5(0) - 8 = A(0-4) + B(0)$

$-8 = -4A$

$A = 2$

$$L^{-1}\left[\frac{5s-8}{s(s-4)}\right] = \frac{2}{s} + \frac{3}{s-4}$$

$$= \frac{2}{s} + 3\left[\frac{1}{s-4}\right]$$

$$= 2 + 3e^{4t}$$

(v) $\frac{s-5}{s^2+4s+20}$

$L^{-1}\left[\frac{s-5}{s^2+4s+20}\right] =$

$F(s) = \frac{s-5}{s^2+4s+4+16} = \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$

$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16} = \frac{s+2}{(s+2)^2+16} - \frac{7}{(s+2)^2+16}$

$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \cdot \frac{4}{(s+2)^2+4^2}$

$F(t) = e^{-2t}\cos 4t - 7/4\, e^{-2t}\sin 4t$

$= e^{-2t}\left[\cos 4t - 7/4 \sin 4t\right]$

(iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2}$

$= F(s) = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$

$A: \left.\frac{s^2-3s-4}{(s-1)^2}\right|_{s=3} = \frac{3^2-3(3)-4}{(3-1)^2} = -1$

$B: \left.\frac{s^2-3s-4}{s-3}\right|_{s=1} = \frac{1^2-3(1)-4}{1-3} = 3$

$C: \frac{d}{ds}\left[\frac{s^2-3s-4}{s-3}\right]_{s=1} = \frac{(s-3)(2s-3)-[s^2-3s-4]}{(s-3)^2}$

at $s=1$

$\frac{(1-3)(2(1)-3)-[1^2-3(1)-4]}{(1-3)^2} = 2$

$$F(s) = -\frac{1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}$$

$$F(t) = -e^{-3t} + 3te^{t} + 2e^{t}$$

$$= e^{t}[3t+2] - e^{3t}$$ //

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ASSIGNMENT (4)

(1) $(1-x^2)\,d^2y/dx^2 - 2x\,dy/dx + 2y = 0$

$(1-x^2)y^2 - 2xy' + 2y = 0$

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \cdots$

$\left[y^{(2+n)} \cdot (1-x^2) + n\,y^{(1+n)} \cdot (-2x) + \frac{n(n-1)}{2!} y^n \cdot (-2)\right]$
$+ \left[y^{(1+n)} \cdot -2x + n y^n \cdot -2 + [2y^n] = 0\right.$

$(1-x^2)y^{n+2} - 2xn\,y^{n+1} - n(n-1)y^n - 2x\,y^{n+1} - 2n\,y^n$
$+ 2y^n = 0$

let $x = 0$

$y^{n+2} - n(n-1)y^n - 2n\,y^n + 2y^n = 0$

$y^{n+2} + y^n[-n(n-1) - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 + n - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 - n + 2] = 0$

$y^{n+2} = -(y^n) \cdot [-n^2 - n + 2]$

$n = 0 : y^2 = -y^0 \cdot 2 = -2y^0$

$n = 1 : y^3 = -y' \cdot [0] = 0$

$n = 2 : y^4 = -y^2[-4] = 4y^2 = 4(-2y^0) = -8y^0$

$n = 3 : y^5 = -y^3 \cdot [-10] = 10y^3 = 10 \cdot 0 = 0$

$n = 4 : y^6 = -y^4 \cdot [-18] = 18y^4 = 18 \cdot 4 \cdot -2 \cdot y^0$

$n = 5 : y^7 = -y^5 \cdot [-28] = 28(y^5) = 28 \cdot 0 = 0$

$y = y^0 + xy' + \frac{x^2 y^3}{2!} + \frac{x^3}{3!} y^3 + \cdots$

$y = y^0 + xy' + \frac{x^2}{2!}(-2)y^0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)y^0 + \cdots$

$\cdots + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(18)4(-2)y^0 + \frac{x^7}{7!}(0)$

$$y = y^{\circ} + x y' - x^2 y^{\circ} - \frac{x^4}{3\times 1} y^{\circ} - \frac{x^6}{5} y^{\circ}$$

$$y = y^{\circ}\left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}\right] + y'_{\circ}[x]$$

2i) $3e^{-4t} - 5e^{4t}$

$$= L[3e^{-4t} - 5e^{4t}] \Rightarrow L[3e^{-4t}] - L[5e^{4t}]$$

$$= 3\left[\frac{1}{s-a}\right] - 5\left[\frac{1}{s-a}\right]$$

$$= 3\left[\frac{1}{s+4}\right] - 5\left[\frac{1}{s-4}\right]$$

$$= \frac{3}{s+4} - \frac{5}{s-4}$$

ii) $\sin 4t + \cos 4t$

$$L[\sin 4t + \cos 4t] = L[\sin 4t] + L[\cos 4t]$$

$$= \frac{a}{s^2+a^2} + \frac{s}{s^2+a^2}$$

$$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$$

$$= \frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{(s^2+16)}$$

iii) $t^3 + 2t^2 - t + 4$

$$t^n = \frac{n!}{s^{n+1}}$$

$$= \frac{3!}{s^{3+1}} + 2\left[\frac{2!}{s^{2+1}}\right] - \left[\frac{1!}{s^{1+1}}\right] + \frac{4}{s}$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

iv) $e^{-2t}\cos 5t$

$$L[\cos 5t] = \frac{s}{s^2+a^2}$$

$$= \frac{s}{s^2+5^2} = \frac{s}{s^2+25}$$

$L[e^{-2t}\cos 5t] = \frac{s+2}{[s+2]^2+25}$

(v) $t \sin 3t$

$L[\sin 3t] = \frac{a}{s^2+a^2}$

$= \frac{3}{s^2+3^2} = \frac{3}{s^2+9}$

$L[t \sin 3t] = -F'(s)$

$\frac{v\, du/ds - u\, dv/ds}{v^2}$

$u = 3$ $\quad du/ds = 0$

$v = s^2+9$ $\quad dv/ds = 2s$

$= \frac{[s^2+9].0 - 3(2s)}{[s^2+9]^2}$

$= \frac{-6s}{[s^2+9]^2}$

$-F'(s) = -1.\left[\frac{-6s}{[s^2+9]^2}\right]$

$= \frac{6s}{[s^2+9]^2}$

(vi) $\frac{e^{-t} - e^{-2t}}{t}$

$L\left[\frac{e^{-t}-e^{-2t}}{t}\right]$

$L[f(t)] = e^{-t} - e^{-2t} = \frac{1}{s+a} - \frac{1}{s+a}$

$= \frac{1}{s+1} - \frac{1}{s+2}$

$F\sigma = \frac{1}{\sigma+1} - \frac{1}{\sigma+2}$

$\int F\sigma \quad L\left[\frac{f(t)}{t}\right] = \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+2}\, d\sigma$

$= \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1}\, d\sigma - \int_{\sigma=s}^{\infty} \frac{1}{\sigma+2}\, d\sigma$

$\ln[\sigma+1] - \ln[\sigma+2)]_s^\infty$

$= [\ln(\sigma+1) - \ln(\sigma+2)]_s^\infty$

$= \left[\ln \frac{\sigma+1}{(\sigma+2)}\right]_s^\infty = \ln\left[\frac{\alpha+1}{(\alpha+2)} - \frac{s+1}{(s+2)}\right]$

$= -\ln\left[\frac{s+1}{(s+2)}\right] = \ln\left[\frac{(s+2)}{s+1}\right]$

vii) $e^{4t}\cos 2t$

$L[\cos 2t] = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$

$L[e^{4t}\cos 2t] = \frac{s-4}{[s-4]^2+4}$

viii) $t\sin 2t$

$L[t\sin 2t] = -\frac{d}{ds}[F(s)]$

$F(s) = L[\sin 2t] = \frac{2}{s^2+2^2}$

$F(s) = \frac{2}{s^2+4}$

$u=2 \qquad du/ds = 0$

$v = s^2+4 \qquad dv/ds = 2s$

$$\frac{v\frac{du}{ds} - u\,dv/ds}{v^2}$$

$$= \frac{(s^2+4)\cdot 0 - 2\cdot(2s)}{(s^2+4)^2}$$

$$= \frac{-4s}{(s^2+4)^2}$$

$L[t\sin 2t] = f'(s)$

$= -1\cdot\left[\frac{-4s}{(s^2+4)^2}\right]$

$= \frac{4s}{[s^2+4]^2}$ //

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(ix) $t^3 + 4t^2 + 5$

$L[t^3 + 4t^2 + 5]$

$= \frac{3!}{s^{3+1}} + 4\left[\frac{2!}{s^{2+1}}\right] + \frac{5}{s}$

$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

x) $e^{3t}(t^2+4)$

let $x = t^2 + 4$

$L[e^{3t} x]$

$L[x] = L[t^2 + 4]$

$= L[t^2] + L[4]$

$= \frac{2!}{s^{2+1}} + \frac{4}{s}$

$= \frac{2}{s^3} + \frac{4}{s}$

$L[e^{3t}] = \frac{2}{[s-3]^3} + \frac{4}{[s-3]}$

xi) $t^2 \cos t$

$L[t^2 \cos t] = t^2 L[\cos t]$

$F(s) = L[\cos t] = \frac{s}{s^2+1^2}$

$F(s) = \frac{s}{s^2+1^2}$

$F'(s) =$ $\quad du/ds = 1$

$u = s$ $\quad dv/ds = 2s$

$v = s^2 + 1^2$

$= \frac{[s^2+1^2] \cdot 1 - 2s[s]}{[s^2+1^2]^2}$

$= \frac{s^2+1^2-2s^2}{[s^2+1^2]^2} = \frac{-s^2+1}{[s^2+1^2]^2}$

$$-f''(s) = -\frac{d}{ds}\left[\frac{s^2-1}{[s^2+1^2]^2}\right]$$

$u = s^2 - 1 \qquad du/ds = 2s$

$v = [s^2+1]^2 \qquad dv/ds = 4s[s^2+1]$

$$\frac{[s^2+1]^2 \cdot 2s - [s^2-1][4s^3+4s]}{[s^2+1]^2}$$

$$= \frac{[2s^5 - 4s^3 + 2s] - [4s^5 - 4s]}{[(s^2+1)^2]}$$

$$= \frac{2s^5 - 4s^3 + 2s - 4s^5 + 4s}{[(s^2+1)^2]}$$

$$= \frac{-2s^5 - 4s^3 + 6s}{[(s^2+1)^2]}$$

$$= \frac{-2s^5 - 4s^3 + 6s}{s^4 + 2s^2 + 1}$$

$$f''(s) = -\frac{d}{ds}\left[\frac{-2s^5 - 4s^3 + 6s}{s^4 + 2s^2 + 1}\right]$$

$$f''(s) = \frac{2s^5 + 4s^3 - 6s}{s^4 + 2s^2 + 1}$$