OHAJI JUDE-THADDEUS

COMPUTER ENG. 13/[illegible]

(1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v'' + \frac{n(n-1)(n-2)}{[illegible]} u^{n-3} v'''$

$+ (1-x^2)y'' - 2xy' + 3y = 0$

For $(1-x^2)y''$, $v = 1-x^2$ $v' = -2x$ $v'' = -2$

$u^n = y^{n+2}$

$(1-x^2)y^{n+2} - ny^{n+2-1}(-2x) + \frac{n(n-1)}{2} y^{n+2-2}(-2) + 0$

$= (1-x^2)y^{n+2} + 2xny^{n+1} - n(n-1)y^n$

For $-2xy$

$v = -2x$, $v' = -2$, $v'' = 0$

$u^n = y^{n+1}$

$= -2xy^{n+1} + ny^{n+1-1}(-2) + 0$

$= -2xy^{n+1} - 2ny^n$

For $2y$

$v = 2$, $v' = 0$

$y^n = u^n$

$= 2y^n$

$y^n = (1-x^2)y^{n+2} + 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n +$

at $x = 0$

$y^{n+2} + 2y^n - (n^2+n)y^n - 2ny^n$

$y^{n+2} = (n^2+n)y^n + 2ny^n - 2y^n$

$y^{n+2} = y^n(n^2+3n-2)$

$(y^{n+2})_0 = (y^n)_0 (n^2+3n-2) \quad n \geq 0$

$n=0$

$(y^2)_0 = -2(y)_0$

$n=1$

$(y^3)_0 = 3(y')_0$

$n=2, \ (y^4)_0 = 8(y^2)_0$

$n=3, \ (y^5)_0 = 48(y')_0$

$n=4, \ (y^6)_0 = 416(y)_0$

$n=5, \ (y^7)_0 = 1824(y')_0$

$y = y_0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3) + \frac{x^4}{4!}(y^4) + \frac{x^5}{5!}(y^5)$

$y = y_0 + x(y') + \frac{x^2}{2}\cdot -2(y) + \frac{x^3}{6}\, 3(y') + \frac{x^4}{24} -16(y) + \frac{x^5}{120} - 1824$

$y = (y_0)_0\left(1 + x^2 - \frac{2}{3}x^4\right) + (y')_0\left[x + \frac{x^3}{2} + \frac{76}{5}x^5 +\right]$

(2)(a) $3e^{-4t} - 5e^{4t}$

$= 3 \cdot \frac{1}{s+4} - 5 \cdot \frac{1}{s-4}$

$= \frac{3}{s+4} - \frac{5}{s-4}$

(b) $\sin 4t + \cos 4t$

$= \frac{4}{s^2+4^2} + \frac{5}{s^2+4^2} = \frac{4}{s^2+16} + \frac{5}{s^2+16}$

(c) $t^3 + 2t^2 - t + 4$

$\frac{3!}{s^4} + 2\left(\frac{2!}{s^3}\right) - \frac{1}{s^2} + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

d) $e^{-2t} \cos 5t$

$= \frac{(s+a)}{(s+a)^2+a^2} = \frac{s+2}{(s+2)^2+5^2} = \frac{s+2}{(s+2)^2+25}$

(e) $t \sin 3t$

$= \frac{-d}{ds}\left(\frac{a}{s^2+a^2}\right) = \frac{-d}{ds}\left(\frac{3}{s^2+9}\right)$

$\frac{d}{ds}\left(3\,(s^2+9)^{-1}\right) = 3\frac{d}{ds}(s^2+9)^{-1}$

$\frac{d}{ds} = \frac{(s^2+a)0 - 3(2s)}{(s^2+9)^2} = \frac{-9s}{(s^2+9)^2}$

(f) $\frac{e^{-t} - e^{2t}}{t}$

$L(e^{-t}) = \frac{1}{s+1}$; $L(-e^{-2t}) = \frac{1}{s+2}$

$$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = (-1)^{-1} \frac{d}{ds}\left[\frac{1}{s^2+3s+2}\right]$$

$$= \frac{2s+3}{(s^2+3s+2)^2}$$

(g) $e^{4t} \cos 2t$

$$= \frac{s-4}{(s-4)^2+4}$$

(h) $t \sin 2t$

$$L(t \sin 2t) = -\frac{d}{ds}\left(\frac{2}{s^2+4}\right) = 2\frac{d}{ds}(s^2+4)^{-1}$$

$$\frac{d}{ds}\left(\frac{2}{s^2+4}\right) = \frac{(s^2+4)0 - 2(2s)}{(s^2+4)^2}$$

$$= \frac{-4s}{(s^2+4)^2}$$

(i) $t^3 + 4t^2 + 5$

$$= \frac{3!}{s^4} + 4\left(\frac{2!}{s^3}\right) + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

(ii) $(e^{3t} \cdot t^2) + 4$

$= \frac{2!}{(s+4)^3} + \frac{4}{s}$

$= \frac{2}{(s-3)^3} + \frac{4}{s}$

(iv) $t^2 \cos t$

$L(t^2 \cos t) = (-1)^2 \frac{d}{ds} \left(\frac{s}{s^2+1} \right)$

$= \frac{(s^2+1) - s(2s)}{(s^2+1)^2} = \frac{1-s^2}{(s^2+1)^2}$

2) $\frac{\sin h 2t}{t}$

$= -1 \left[\frac{4s}{(s^2-4)^2} \right] = \frac{4s}{(s^2-4)^2}$

3(i) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$\frac{s-5}{(s-3)(s-4)} = \frac{A(s-4) + B(s-3)}{(s-3)(s-4)}$

$s-5 = A(s-4) + B(s-3)$

$s=4$,

$4-5 = A(4-4) + B(4-3)$

$B = -1$

$s = 3$

$3-5 = A(3-4)$

$A = 2$

$= \frac{2}{s-3} - \frac{1}{s-4} = 2\left(\frac{1}{s-3}\right) - \frac{1}{s-4}$

$= 2e^{3t} - e^{4t}$

(ii) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$\frac{2s-6}{(s-2)(s-4)} \quad \frac{A(s-4)+B(s-2)}{(s-2)(s-4)}$

$2s-6 = A(s-4)+B(s-2)$

$s = 2,$

$A = 1$

$s = 4$

$2 = 2B$

$B = 1$

$= \frac{1}{s-2} + \frac{1}{s-4} = e^{2t} + e^{4t}$

(iii) $\frac{5s-8}{s(s-4)}$

$= A/s + B/(s-4)$

$$\frac{5s-8}{s(s-4)} = \frac{A(s-4)+Bs}{s(s-4)}$$

$$5s-8 = A(s-4)+Bs$$

$s=0$ $s=4$

$\frac{-8}{-4} = \frac{-4A}{-4}$ $\frac{12}{4} = \frac{4B}{4}$

$A=2$ $B=3$

$$= \frac{2}{s} + \frac{3}{s-4}$$

$$= 2 + 3e^{4t}$$

) $\frac{(s^2-3s-4)}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$

$$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A(s-1)^2+B(s-3)(s-1)+C(s-3)}{(s-3)(s-1)^2}$$

$$\frac{s^2-3s-4}{\cancel{(s-3)(s-1)^2}} = A(s-1)^2+B(s-3)(s-1)+C(s-3)$$

$s=3$

$$3^2-3(3)-4 = A(3-1)^2$$

$\frac{-4}{4} = \frac{4A}{4}$ $A=-1$

$s=1$

$1^2 - 3(1) - 4 = A(\cancel{s-1}) + B(\cancel{s-3})(s-1) + C(s-3)$

$\cancel{-6}\ 3 = \cancel{-2}\ C$

$\cancel{-2} \qquad \cancel{-2} \qquad C = 3$

$s = 0$

$0^2 - 3(0) - 4 = A(0-1)^2 + B(0-3)(0-1) + C(0-3)$

$-4 = A + 3B - 3C$

$-4 = -1 + 3B - 3(3)$

$-4 = -1 + 3B - 9$

$-4 = -10 + 3B$

$-4 + 10 = 3B \qquad B = 2$

$\frac{6\ 2}{3} = \frac{3B}{\cancel{3}}$

$= \frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}$

$= -e^{3t} + 2e^{t} + 3te^{t}$

(v) $\frac{s-5}{s^2+4s+20}$

$\frac{s-5}{s^2+4s+20} = \frac{As+B}{s^2+4s+20}$

$s-5 = \frac{As + B}{\cancel{s^2+4s+20}}$

$A = 1, \quad B = -5$