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1) Use the Leibnitz - Maclaurin method to determine a series solution for

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

Solution

$(1-x^2)\,d^2y/dx^2 - 2x\,dy/dx + 2y = 0$

$(1-x^2)\,y^2 - 2xy^1 + 2y = 0$

$y^n = u^n v + n u^{n-1} v^1 + \frac{n(n-1)}{2!} u^{n-2} v^2 + \dots +$

$[y^{(2+n)}\cdot(1-x^2) + n y^{(1+n)}\cdot(-2x) + \frac{n(n-1)}{2!} y^n(-2)] +$

$[y^{(1+n)}\cdot -2x + n y^n \cdot -2 + [2y^n] = 0$

$(1-x^2)y^{n+2} - 2x\cdot n y^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n = 0$

let $x = 0$

$y^{n+2} - n(n-)y^n - 2n\cdot y^n + 2y^n = 0$

$y^{n+2} + y^n[-n(n-1) - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 + n - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 - n + 2] = 0$

$y^{n+2} = -(y^n) = [-n^2 - n + 2]$

$n = 0$: $y^2 = -y^0 \cdot 2 = -2y^0$

$n = 1$: $y^3 = 0y^1\cdot[0] = 0$

$n = 2$: $y^4 = -y^2[-4] = 4y^2 = 4[-2y^0] = -8y^0$

$n = 3$: $y^5 = -y^3\cdot[-10] = 10y^3 = 10\cdot 0 = 0$

$n = 4$: $y^6 = -y^4\cdot[-18] = 18y^4 = 18\cdot 4\cdot -2\cdot y^0$

$n = 5$: $y^7 = -y^5\cdot[-28] = 28(y^5) = 28\cdot 0 = 0$

$$y = y^0 + xy^1 + \frac{x^2}{2!}y^2 + \frac{x^3}{3!}y^3 + \dots$$

$$y = y_0 + x y_0' + \frac{x^2}{2!}(-2) y_0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2) y_0 + \dots + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(18)4(-2) y_0 + \frac{x^7}{7!}(0)$$

$$y = y_0 + x y_0' - x^2 y_0 - \frac{x^4}{3\times2\times1} y_0 - \frac{x^6}{5} y_0$$

$$y = y_0\left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}\right] + y_0'[x]$$

2i) $3e^{-4t} - 5e^{4t}$

$\Rightarrow L[3e^{-4t} - 5e^{4t}]$

$$L[3e^{-4t}] = \frac{3}{s-(-4)} = \frac{3}{s+4}$$

$$L[-5e^{4t}] = \frac{5}{s-4} = \frac{5}{s-4}$$

$$L[3e^{-4t} - 5e^{4t}] = \frac{3}{s+4} - \frac{5}{s-4}$$

2ii) $\sin 4t + \cos 4t$

$$L[\sin 4t] = \frac{4}{s^2+4^2} = \frac{4}{s^2+16}$$

$$L[\cos 4t] = \frac{s}{s^2+4^2} = \frac{s}{s^2+16}$$

$$L[\sin 4t + \cos 4t] = \frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{s^2+16}$$

2iii) $t^5 + 2t^2 - t + 4$

$$L[t^5] = \frac{5!}{s^{5+1}} = \frac{5!}{s^6} = \frac{120}{s^6}$$

$$L[2t^2] = 2\left[\frac{2!}{s^{2+1}}\right] = \frac{4!}{s^3} = \frac{24}{s}$$

$$L[t] = \frac{1!}{s^{1+1}} = \frac{1}{s^2}$$

$$L[4] = 4/s$$

$\therefore L[t^5 + 2t^2 - t + 4]$

$$\Rightarrow \frac{120}{s^6} + \frac{24}{s} - \frac{1}{s^2} + \frac{4}{s}$$

$$\Rightarrow \frac{120 + 24s^5 - s^4 + 4s^5}{s^6}$$

iv) $e^{-2t} \cos 5t$

$$L[e^{-2t} \cos 5t] = \frac{s-(-2)}{s^2+5^2} = \frac{s+2}{s^2+25} = \frac{s+2}{(s+2)^2+25}$$

v) $t \sin 3t$

$$L[t \sin 3t] = \frac{a}{s^2+a^2}$$

$$= \frac{3}{s^2+3^2} = \frac{3}{s^2+9}$$

$$L[t \sin 3t] = -f'(s) \qquad \frac{v \, du/ds - u \, dv/ds}{v^2}$$

$u = 3 \quad du/ds = 0$

$v = s^2+9 \quad dv/ds = 2s$

$$= \frac{[s^2+9] \cdot 0 - 3[2s]}{[s^2+9]^2} = \frac{-6s}{[s^2+9]^2}$$

$$\Rightarrow -f'(s) = -1\left[\frac{-6s}{[s^2+9]^2}\right] = \frac{6s}{[s^2+9]^2}$$

vi) $\dfrac{e^{-t} - e^{-2t}}{t}$

$$L\left[\frac{e^{-t} - e^{-2t}}{t}\right]$$

$$L[f(t)] = e^{-t} - e^{-2t} = 1/s+1 \quad \frac{1}{s+1}$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$f(\sigma) = \frac{1}{\sigma+1} - \frac{1}{\sigma+2}$$

$$\int f\sigma \quad L\left[\frac{f(t)}{t}\right] = \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+2}\, d\sigma$$

$$= \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1}\, d\sigma - \int_{\sigma=s}^{\infty} \frac{1}{\sigma+2}\, d\sigma$$

$$\ln[\sigma+1] - \ln[\sigma+2]_s^{\infty}$$

$$= \left[\ln(\sigma+1) - \ln(\sigma+1)\right]_s^{\infty}$$

$$= \left[\ln \frac{\sigma+1}{(\sigma+2)}\right]_{\sigma}^{\infty} = \ln\left[\frac{\alpha+1}{(\alpha+2)} - \frac{s+1}{(s+2)}\right]$$

$$= -\ln\left[\frac{\sigma+1}{(s+2)}\right] = \ln\left[\frac{(s+2)}{s+1}\right]$$

v(ii) $e^{3t}\cos 2t$

$$L[\cos 2t] = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

$$L[e^{4t}\cos 2t] = \frac{s-4}{[s-4]^2+4}$$

v(iii) $t\sin 2t$

$$L[t\sin 2t] = -\frac{d}{ds}[f(s)]$$

$$f(s) = L[\sin 2t] = 2/s^2+2^2$$

$$f(s) = \frac{2}{s^2+4}$$

$$u = 2 \qquad du/ds = 0$$

$$v = s^2+4 \qquad dv/ds = 2s$$

$$\frac{v\, du/ds - u\, du/ds}{v^2}$$

$$= \frac{(s^2+4)\cdot 0 - 2\cdot(2s)}{(s^2+4)^2}$$

$$= \frac{-4s}{(s^2+4)^2}$$

$$L[t\sin 2t] = f'(s)$$

$$= -1\cdot\left[\frac{-4s}{(s^2+4)^2}\right] = \frac{4s}{(s^2+4)^2}$$

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1)

$t^3 + 4t^2 + 5$

$$L[t^3 + 4t^2 + 5] = \frac{3!}{s^{3+1}} + 4\left[\frac{2!}{s^{2+1}}\right] + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

x

$e^{3t}(t^2+4)$

let $x = t^2 + 4$

$L[e^{3t}x]$

$$L[x] = L[t^2+4]$$

$$= L[t^2] + L[4]$$

$$= \frac{2!}{s^{2+1}} + \frac{4}{s}$$

$$= \frac{2}{s^3} + \frac{4}{s}$$

$$L[e^{3t}] = \frac{2}{[s-3]^3} + \frac{4}{[s-3]}$$

xi)

$t^2 \cos t$

$$L[t^2 \cos t] = t^2 L[\cos t]$$

$$F(s) = L[\cos t] = \frac{s}{s^2+1^2}$$

$$f(s) = \frac{s}{s^2+1^2}$$

$f'(s) = du/ds = 1$

$u = s \quad dv/ds = 2s$

$v = s^2 + 1^2$

$$= \frac{[s^2+1^2] \cdot 1 - 2s[s]}{[s^2+1^2]^2}$$

$$= \frac{s^2+1^2-2s^2}{[s^2+1^2]} = \frac{-s^2+1}{[s^2+1^2]^2}$$

$$-f''(s) = \frac{-d}{ds}\left[\frac{s^2-1}{[s^2+1^2]^2}\right]$$

$u = s^2 - 1 \qquad du/ds = 2s$

$v = [s^2+1]^2 \qquad dv/ds = 4s[s^2+1]$

$$= \frac{[s^2+1]^2 \cdot 2s - [s^2-1][4s^3+4s]}{[s^2+1]^2}$$

$$= \frac{[2s^5 - 4s^3 + 2s] - [4s^5 - 4s]}{[(s^2+1)^2]}$$

$$= \frac{2s^5 - 4s^3 + 2s - 4s^5 + 4s}{[(s^2+1)^2]}$$

$$= \frac{-2s^5 - 4s^3 + 6s}{[(s^2+1)^2]}$$

$$= \frac{-2s^5 - 4s^2 + 6s}{s^4+2s^2+1}$$

$$f''(s) = \frac{-d}{ds}\left[\frac{-2s^5 - 4s^3 + 6s}{s^4+2s^2+1}\right]$$

$$f''(s) = \frac{2s^5 + 4s^3 - 6s}{s^4+2s^2+1}$$

Question 3

3.) $\frac{s-5}{(s-3)(s-4)}$

$$L^{-1}\left[\frac{s-5}{(s-3)(s-4)}\right] = \frac{A}{(s-3)} + \frac{B}{(s-4)}$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A(s-4) + B(s-3)}{(s-3)(s-4)}$$

Assuming $s = 4$

$4 - 5 = A(4-4) + B(4-3)$

$-1 = B(1)$

$B = -1$.

Assuming $s = 3$

$3 - 3 = A(3-4) + B(3-3)$

$$-2 = A(-1)$$
$$A = 2$$
$$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} + \frac{-1}{s-4}$$
$$= \frac{2}{s-3} - \frac{1}{s-4}$$
$$= 2\left[\frac{1}{s-3}\right] - \left[\frac{1}{s-4}\right]$$
$$= 2e^{3t} - e^{4t}$$

iv) $\frac{2s-6}{(s-2)(s-4)}$

$$L^{-1}\left[\frac{2s-6}{(s-2)(s-4)}\right]$$
$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$$
$$2s-6 = A(s-4) + B(s-2)$$
Assuming $s = 4$
$$2(4) - 6 = A(4-4) + B(4-2)$$
$$8-6 = A(0) + B(2)$$
$$2 = 2B$$
$$B = 1$$
$s = 2$
$$2(2) - 6 = A(2-4) + B(2-2)$$
$$4-6 = -2A + 0$$
$$-2 = -2A$$
$$A = 1$$
$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$
$$L^{-1}\left[\frac{2s-6}{(s-2)(s-4)}\right] = e^{2t} + e^{4t}$$

iv)

$$\frac{5s-8}{s(s-4)}$$

$$L^{-1}\left[\frac{5s-8}{s(s-4)}\right] = \frac{A}{s} + \frac{B}{s-4}$$

$$5s-8 = A(s-4) + B(s)$$

Assume $s=4$

$$5(4)-8 = A(4-4) + B(4)$$

$$20-8 = 4B$$

$$12 = 4B$$

$$B = 3$$

Assuming $s=0$

$$5(0)-8 = A(0-4) + B(0)$$

$$-8 = -4A$$

$$A = 2$$

$$L^{-1}\left[\frac{5s-8}{s(s-4)}\right] = \frac{2}{s} + \frac{3}{s-4}$$

$$= \frac{2}{s} + 3\left[\frac{1}{s-4}\right]$$

$$= 2 + 3e^{4t}$$

v)

$$\frac{s-5}{s^2+4s+20}$$

$$L^{-1}\left[\frac{s-5}{s^2+4s+20}\right] = f(s) = \frac{s-5}{s^2+4s+4+16}$$

$$\Rightarrow \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16} = \frac{s+2}{(s+2)^2+16} - \frac{2}{(s+2)^2+1}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \cdot \frac{4}{(s+2)^2+4^2}$$

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$$f(t) = e^{-2t}\cos 4t - \tfrac{7}{4}e^{-2t}\sin 4t$$
$$= e^{-2t}\left[\cos 4t - \tfrac{7}{4}\sin 4t\right]$$

iv) $\dfrac{s^2-3s-4}{(s-3)(s-1)^2}$

$$= f(s) = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$$

$$A: \left.\frac{s^2-3s-4}{(s-1)^2}\right|_{s=0} = \frac{3^2-3(3)-4}{(3-1)^2} = -1$$

$$B: \left.\frac{s^2-3s-4}{(s-3)}\right|_{s=1} = \frac{1^2-3(1)-4}{1-3} = 3$$

$$C: \frac{d}{ds}\left[\frac{s^2-3s-4}{s-3}\right]_{s=1} = \frac{(s-3)(2s-2)-[s^2-3s-4]}{(s-3)^2}$$

at $s=1$

$$\frac{(1-3)(2(1)-3)-[1^2-3(1)-4]}{(1-3)^2} = 2$$

$$f(s) = \frac{-1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}$$

$$f(t) = -e^{-3t} + 3te^{t} + 2e^{t}$$
$$= e^{t}[3t+2] - e^{3t}$$